Spintronics II

1. Valet-Fert model for CPP GMR and spin accumulation
2. Tunneling magnetoresistance
3. New trends in spintronics: the emerging field of spin-orbitronics

Italian School of Magnetism

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Outlook

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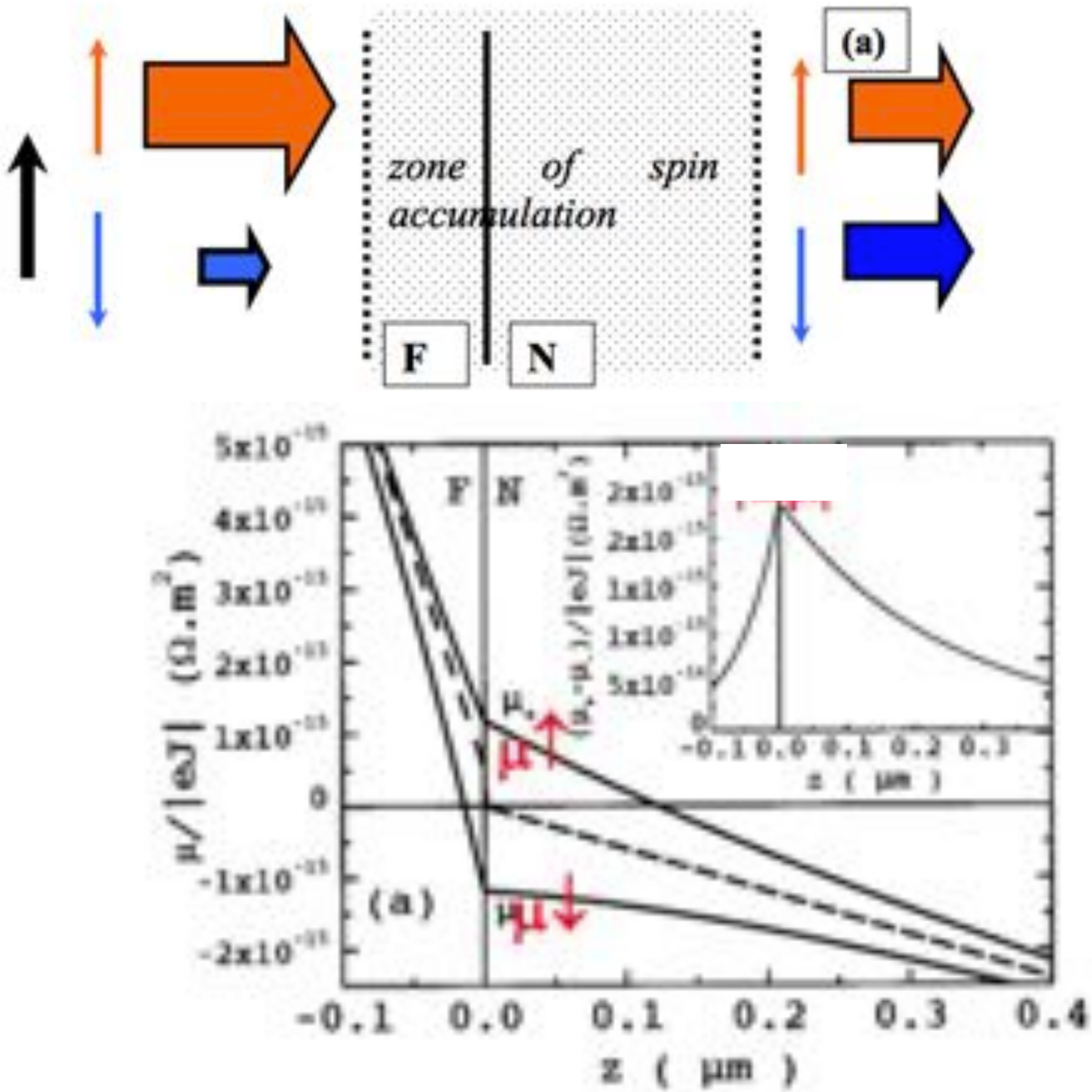
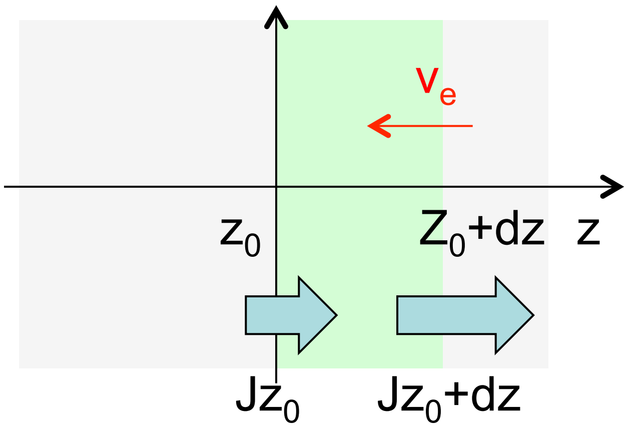
Spin-accumulation



Co/cu interface:
$\beta = 0.46 > 0$
$\rho+ < \rho-$

In the bulk of Co
J+ > J-

Equilibrium condition



$$\nabla \cdot J_{\uparrow} = -\frac{\partial \rho_{\uparrow}}{\partial t} = -\frac{\partial\left(-en_{\uparrow}\right)}{\partial t} = e\frac{\partial n_{\uparrow}}{\partial t}$$

If divJ>0 There's an electron accumulation in the green zone: non stationary case!

To find a stationary state the excess of spin up electrons entering from the right side must undergo a spin flip towards the spin down channel. This corresponds to the spin flip term added to the Boltzmann equation:

$$\left(\frac{\partial f_{\uparrow}}{\partial t}\right)_{sf} = -\frac{f_{\uparrow}(\mathbf{v}) - f_{\downarrow}(\mathbf{v})}{\tau_{sf}}$$

Spin-flip rate towards the spin up channel from the spin down channel

$$\frac{\partial n_{\uparrow}}{\partial t} = \underbrace{\frac{1}{e}(\nabla \cdot J_{\uparrow})}_{\text{Acc. rate due to currents}} - \underbrace{\frac{n_{\uparrow} - n_{\downarrow}}{\tau_{sf}}}_{\text{Acc. rate due to spin-flip}} = 0$$

Time relaxation approximation

$$\underbrace{\left(\frac{\partial f_s}{\partial t}\right)_{coll} = -\frac{f_s(\mathbf{v}) - f_0(\mathbf{v})}{\tau}}_{\text{Scattering without spin flip}} \qquad \underbrace{\left(\frac{\partial f_\uparrow}{\partial t}\right)_{sf} = -\frac{f_\uparrow(\mathbf{v}) - f_\downarrow(\mathbf{v})}{\tau_{sf}}}_{\text{Spin flip}}$$

Local mean free path

$$\lambda_S = v_F\left(\frac{1}{\tau_s} + \frac{1}{\tau_{sf}}\right)^{-1}$$

The scattering probabilities are additive

Spin diffusion length

$$l_s = \left(D_s \tau_{sf}\right)^{1/2}$$

D_s : diffusion constant for channel s

T. Valet, A. Fert, *Phys. Rev. B* **1993**, *48*, 7099

Macroscopic transport equations

$\bar{\mu}_s(z) = \mu_s(z) - eV(z)$ Electrochemical potential for spin s

If $\lambda_s << l_{sf}$ the Boltzmann equation leads to:

(1) $$\frac{e}{\sigma_s}\frac{\partial J_s}{\partial z} = \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2},$$ Change in J_s due to spin flip

(2) $$J_s = \frac{\sigma_s}{e}\frac{\partial \bar{\mu}_s}{\partial z}$$ Ohm's law

Meaning of eq. (1):

$$e\rho_\uparrow \frac{\partial J_\uparrow}{\partial z} = \frac{\mu_\uparrow - \mu_\downarrow}{l_\uparrow^2}$$

$$e\frac{m}{n_\uparrow e^2 \tau_\uparrow}\frac{\partial J_\uparrow}{\partial z} = \frac{\mu_\uparrow - \mu_\downarrow}{\frac{1}{3} v_F^2 \tau_\uparrow \tau_{sf}}$$

$$\frac{\partial J_\uparrow}{\partial z} = \left(\frac{3}{2}\frac{n_\uparrow}{\frac{1}{2} m v_F^2}\right) e \frac{\mu_\uparrow - \mu_\downarrow}{\tau_{sf}} = e\frac{N_\uparrow(E_F)(\mu_\uparrow - \mu_\downarrow)}{\tau_{sf}}$$ Free electrons approx.

$$\frac{\partial J_\uparrow}{\partial z} = e\frac{n_\uparrow - n_\downarrow}{\tau_{sf}}$$ Spin flip processes balance the div **J**

To solve the transport equations: $\bar{\mu}_{\pm} = \bar{\mu} \pm \Delta\mu$

The gradient of part of the chemical potential independent on the spin is the equivalent of an electric field:

$$F(z) = \frac{1}{e}\frac{\partial \bar{\mu}}{\partial z}$$

The transport equations become:

$$\frac{e}{\sigma_s}\frac{\partial J_s}{\partial z} = \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2},$$

$$J_s = \frac{\sigma_s}{e}\frac{\partial \bar{\mu}_s}{\partial z}$$

$$\frac{e}{\sigma_{\pm}}\frac{\partial J_{\pm}}{\partial z} = \pm 2\frac{\Delta\mu}{l_s^2}, \qquad (3)$$

$$J_{\pm}(z) = \sigma_{\pm}\left[F(z) \pm \frac{1}{e}\frac{\partial \Delta\mu}{\partial z}\right] \qquad (4)$$

Put (4) in (3):

$$e\left[\frac{\partial F(z)}{\partial z} + \frac{1}{e}\frac{\partial^2 \Delta\mu}{\partial z^2}\right] = \frac{2\Delta\mu}{l_{\uparrow}^2}$$

$$e\left[\frac{\partial F(z)}{\partial z} - \frac{1}{e}\frac{\partial^2 \Delta\mu}{\partial z^2}\right] = -\frac{2\Delta\mu}{l_{\downarrow}^2}$$

Average spin diffusion length:

$$\left(\frac{1}{l_{sf}}\right)^2 = \left(\frac{1}{l_{\uparrow}^2} + \frac{1}{l_{\downarrow}^2}\right)$$

By subtracting the two equations:

$$\frac{\partial^2 \Delta\mu}{\partial z^2} = \Delta\mu\left(\frac{1}{l_{\uparrow}^2} + \frac{1}{l_{\downarrow}^2}\right)$$

Thus:

$$\frac{\partial^2 \Delta\mu}{\partial z^2} = \frac{\Delta\mu}{l_{sf}^2}$$

Charge conservation:

$$\frac{\partial}{\partial z}\left(J_{+}+J_{-}\right)=0 \qquad \frac{\partial^2}{\partial z^2}(\sigma_{+}\bar{\mu}_{+}+\sigma_{-}\bar{\mu}_{-})=0$$

We must solve these equations :

$$\begin{cases} \dfrac{\partial^2 \Delta\mu}{\partial z^2}=\dfrac{\Delta\mu}{l_{sf}^2} \\ \dfrac{\partial^2}{\partial z^2}(\sigma_{+}\bar{\mu}_{+}+\sigma_{-}\bar{\mu}_{-})=0 \end{cases}$$

The solution in a homogeneous medium is:

$$\Delta\mu = A\exp(z/l_{sf})+B\exp(-z/l_{sf})$$

$$(\sigma_{+}\bar{\mu}_{+}+\sigma_{-}\bar{\mu}_{-})=Cz+D\ .$$

The volume resistivity can be written as:

$$\rho_{\uparrow(\downarrow)}=1/\sigma_{\uparrow(\downarrow)}=2\rho_F^*[1-(+)\beta] \qquad \text{FM}$$

$$\rho_{\uparrow(\downarrow)}=2\rho_N^* \qquad \text{Not FM (N)}$$

Isolated interface between two FM materials

(A) | (B)

$$\rho_\pm = 2\rho_F^*(1\pm\beta) \qquad\qquad \rho_\pm = 2\rho_F^*(1\mp\beta)$$

An el. + is minority | An el + is majority

Solution in (A)

In (B) we must change all signs.

$$\Delta\mu(z) = \frac{\beta}{1-\beta^2} eE_0 l_{sf}^F \exp\left[\frac{z}{l_{sf}^F}\right],$$

$$F(z) = E_0\left[1+\frac{\beta^2}{1-\beta^2}\exp\left[\frac{z}{l_{sf}^F}\right]\right],$$

$$J_+(z) = (1-\beta)\frac{J}{2}\left[1+\frac{\beta}{1-\beta}\exp\left[\frac{z}{l_{sf}^F}\right]\right],$$

$$J_-(z) = (1+\beta)\frac{J}{2}\left[1-\frac{\beta}{1+\beta}\exp\left[\frac{z}{l_{sf}^F}\right]\right],$$

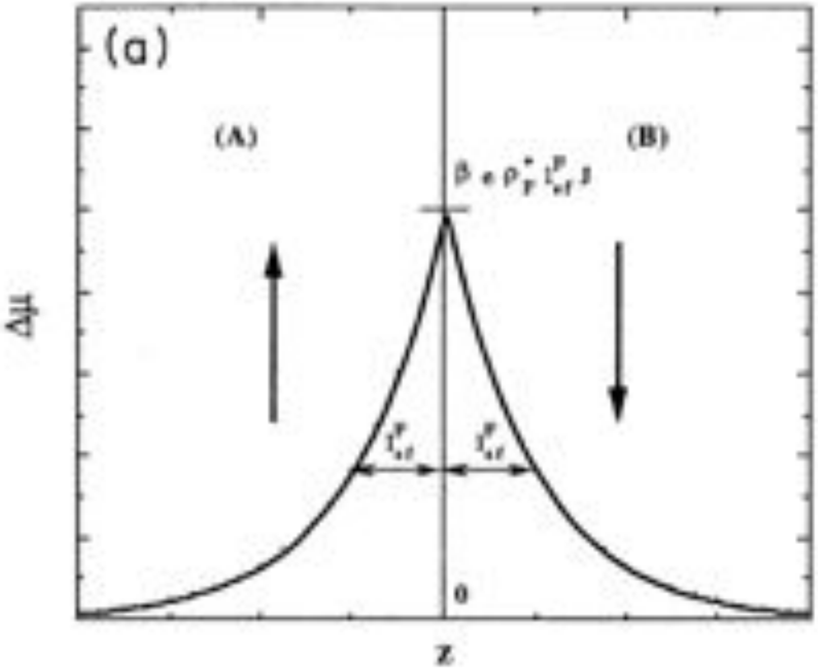
$$\overline{\mu}^\pm(z=0^+) = \overline{\mu}^\pm(z=0^-)$$

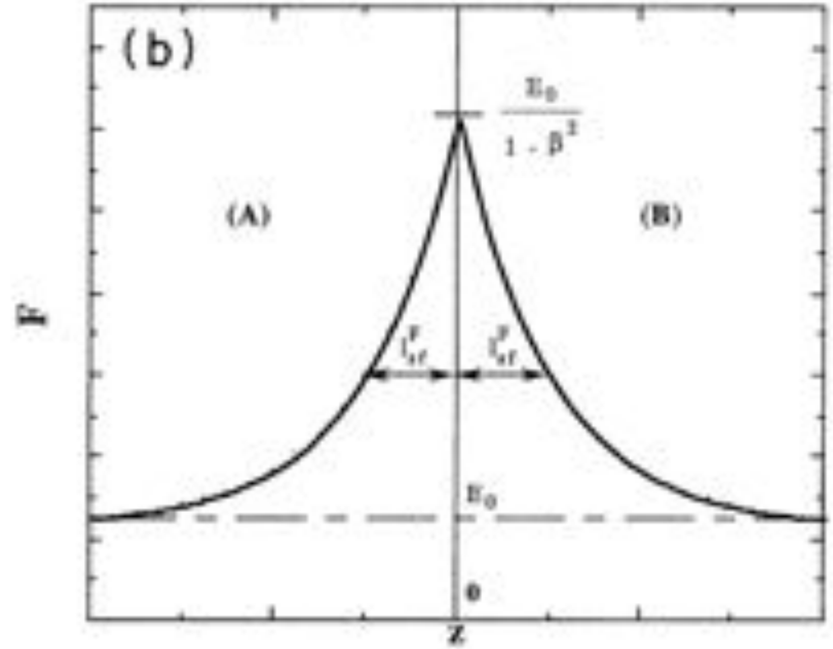
$$J^\pm(z=0^+) = J^\pm(z=0^-)$$

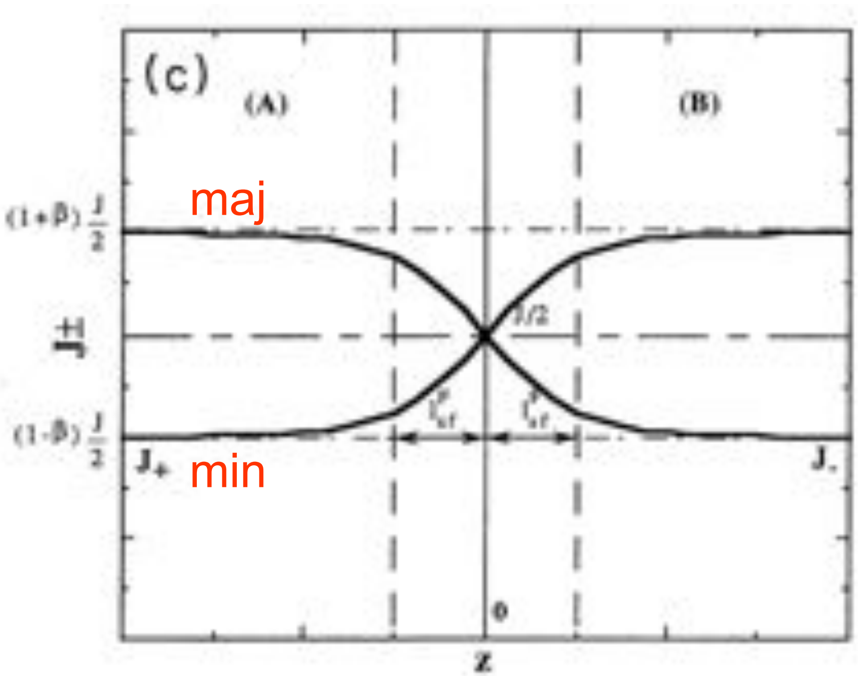
4 variables: K_{1A}, K_{2A}, K_{1B}, K_{3B}

Electric field far away from the interface:

$$E_0 = (1-\beta^2)\rho_F^* J.$$







The AP gives rise to an additional voltage drop with respect to that due to $E_{0:}$

$$\Delta V_I = \int_{-\infty}^{+\infty} [F(z) - E_0] dz = 2\beta^2 \rho_F^* l_{sf}^F J$$

The corresponding interface resistance is (per unit surface):

$$r_{SI} = 2\beta^2 \rho_F^* l_{sf}^F$$

β>0 means $r_\uparrow < r_\downarrow$ ⇨ J↑ > J↓

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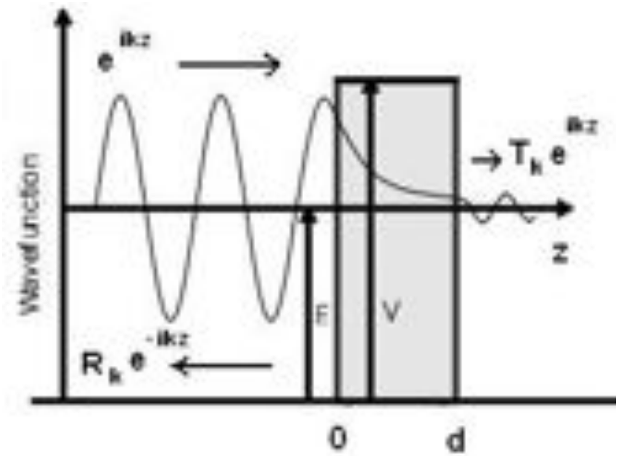
Tunneling and WKB



$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(z)}{\partial z^2}+V\Psi(z)=E_z\Psi(z)$$

$$\begin{cases} \Psi = e^{ik_z z} + R_k e^{-ik_z z} & \text{per} \quad z<0 \\ \Psi = Ae^{-k'(E_z)z} + Be^{k'(E_z)z} & \text{per} \quad 0<z<d \\ \Psi = T_k e^{ik_z z} & \text{per} \quad z>d \end{cases}$$

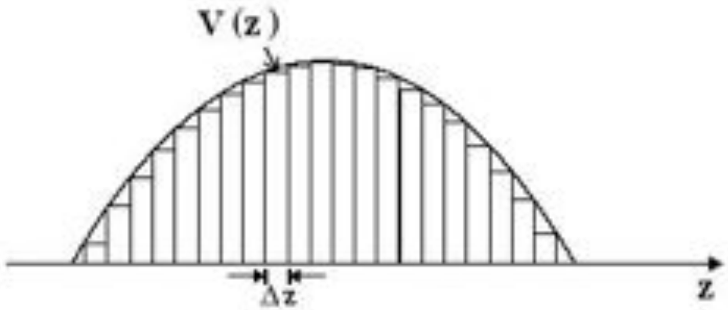
$$k_z = \sqrt{\frac{2mE_z}{\hbar^2}} \qquad k'(E_z) = \sqrt{\frac{2m(V-E_z)}{\hbar^2}}$$

$$|T_k|^2 = \frac{16k^2k'^2}{(k^2+k'^2)^2}e^{-2k'd}$$ for large values of k'd

Consider a non rectangular barrier:

$$\log|T|^2 \approx -2k'd + 2\log\frac{4(kd)(k'd)}{(kd)^2+(k'd)^2}$$ The first term dominates

Approximate the barrier with the series of rectangular barriers
Suppose that transmission coefficients are multiplicative.



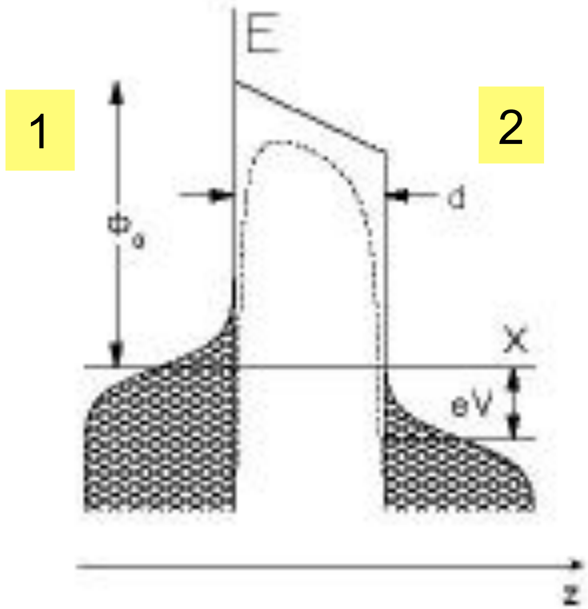
$$\log|T|^2 \approx \sum_{\substack{barriere\\parziali}} \log\left|T_{\substack{barriere\\parziali}}\right|^2 \approx -2\sum \Delta z\langle k'\rangle$$

For Δz→0 :

$$\log|T|^2 \approx -2\int_{barriera} dz\sqrt{\left(2m/\hbar^2\right)\left[V(z)-E\right]}$$

$$|T|^2 \approx e^{-2\int_{barriera} dz\sqrt{\left(2m/\hbar^2\right)\left[V(z)-E\right]}}$$

I-V characteristic of a tunneling junction



$$j = j_{1\to 2} - j_{2\to 1}$$

$$I^{\pm}_{1\to 2}(V,E) = D^{\pm}_{1}(E)f(E)|M|^{2}D^{\pm}_{2}(E+eV)(1-f(E+eV))$$

$$I^{\pm}_{2\to 1}(V,E) = D^{\pm}_{1}(E)(1-f(E))|M|^{2}D^{\pm}_{2}(E+eV)f(E+eV)$$

$$j = \sum_{k}\int_{-\infty}^{+\infty} dE D_{1}(E)D_{2}(E+eV)|M(E)|^{2}(f(E)-f(E+eV))$$

Sum over transversal *k*, *D(E)* is the density of states at the energy E (with respect to the Fermi level) and *f(E)* is the Fermi distribution . The matrix element is proportional to $|T(E)|^2$ calculated with the WKB approximation.

J. G. Simmons, J. Appl. Phys. 34, 1793 (1963)

$$j(V) = \frac{J_0}{d^2}\left(\overline{\Phi} - \frac{eV}{2}\right)\exp\left[-Ad\sqrt{\overline{\Phi} - \frac{eV}{2}}\right] - \frac{J_0}{d^2}\left(\overline{\Phi} + \frac{eV}{2}\right)\exp\left[-Ad\sqrt{\overline{\Phi} + \frac{eV}{2}}\right]$$

$$A = \frac{4\pi}{h}\sqrt{2m_c^*} \qquad J_0 = \frac{e}{2\pi h}$$

d in Angstroms, Φ in V $\qquad$ $A=1.025\ eV^{-1/2}\ Å^{-1}$ $\qquad$ $J_0=6.2 \times 10^{10}\ eV^{-1}\ Å^{2}$

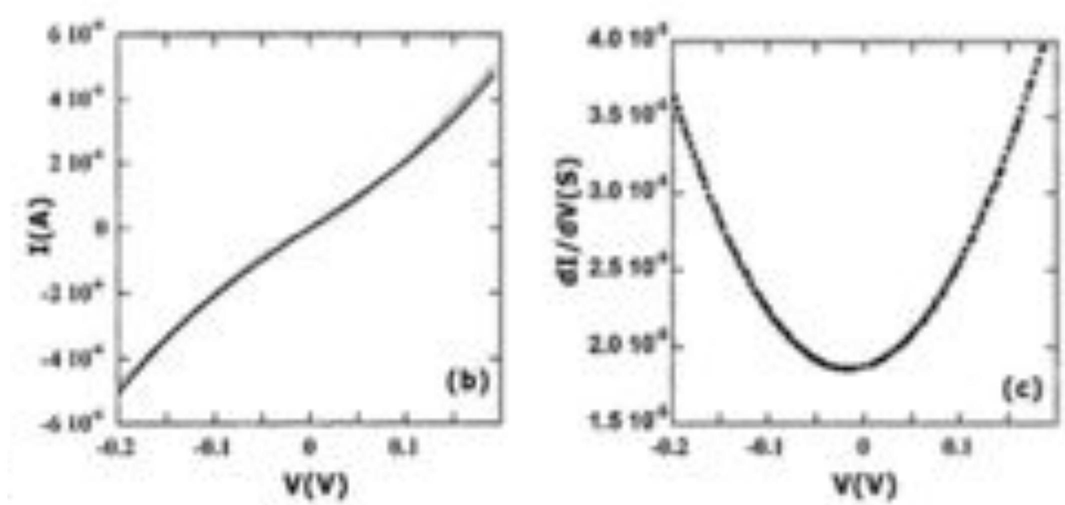
In this wayJ is expressed in A/cm^2

$$j(V)=\frac{J_0}{d^2}\left(\overline{\Phi}-\frac{eV}{2}\right)\exp\left[-Ad\sqrt{\overline{\Phi}-\frac{eV}{2}}\right]-\frac{J_0}{d^2}\left(\overline{\Phi}+\frac{eV}{2}\right)\exp\left[-Ad\sqrt{\overline{\Phi}+\frac{eV}{2}}\right]$$

For small values of the applied voltage:

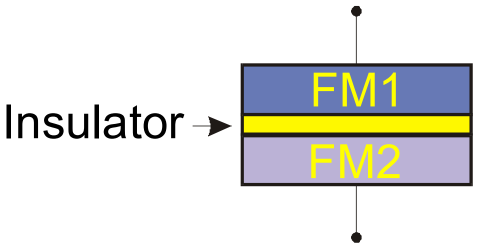
$J \approx \alpha V + \beta V^3$ The conductance G = dI/dV has a parabolic shape

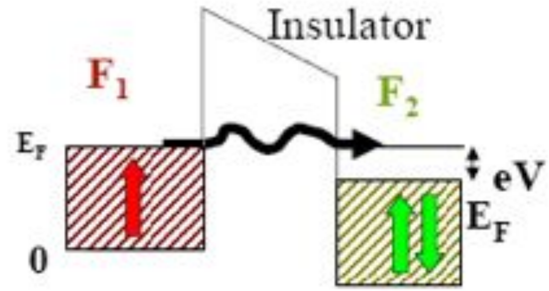
The current depends exponentially on:
- the barrier thickness
- the square root of the barrier height

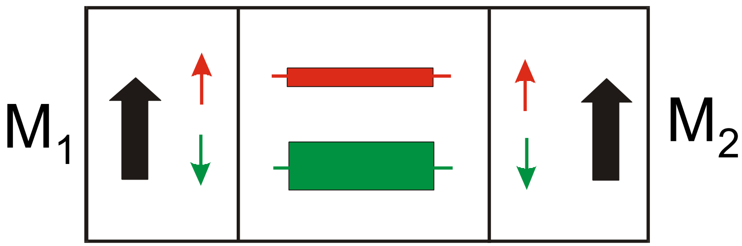


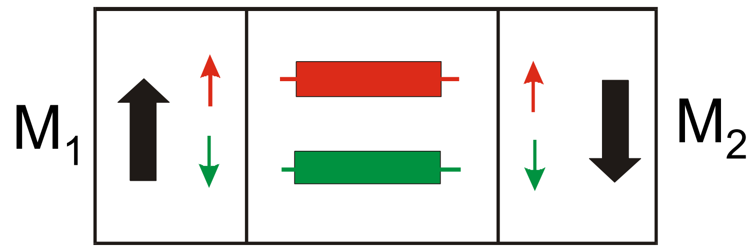
Adapted from: A Barry et al. A CrO2-based magnetic tunnel junction. *J. Phys. Condens. Matter* **12,** L173 (2000).

Spin Dependent Tunneling

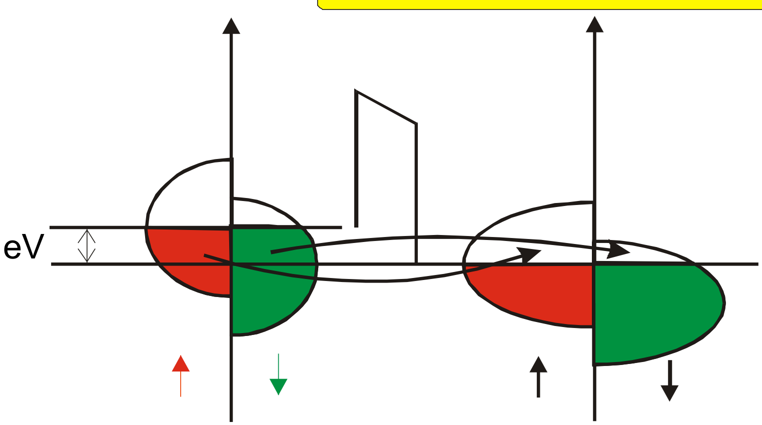


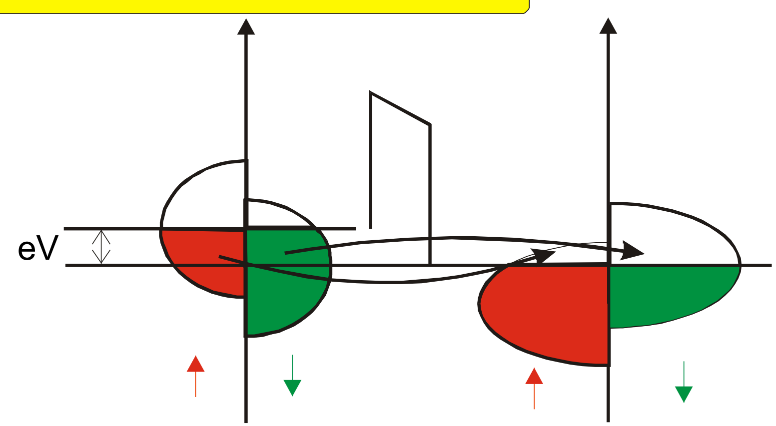




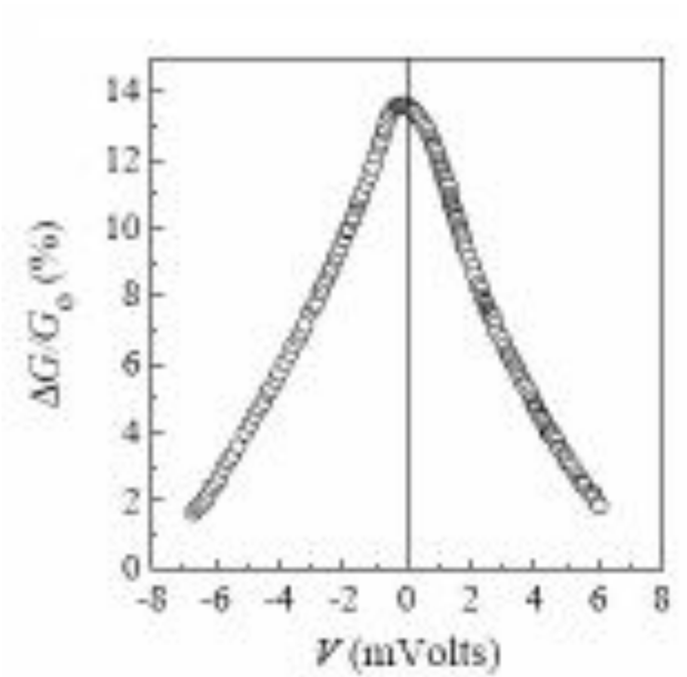


Rp < Rap





Jullière model for TMR (1975)



Fe/GeO$_x$/Co

Assumptions:
- Spin conservation during tunneling
- Constant transmission coefficients, independent on magnetization and energy
- Small applied voltage

$$G_P = G_{\uparrow\uparrow} + G_{\downarrow\downarrow} \propto D_{1\uparrow} D_{2\uparrow} + D_{1\downarrow} D_{2\downarrow}$$

$$G_{AP} = G_{\uparrow\downarrow} + G_{\downarrow\uparrow} \propto D_{1\uparrow} D_{2\downarrow} + D_{1\downarrow} D_{2\uparrow}$$

$$P_i = \frac{D_{1\uparrow} - D_{1\downarrow}}{D_{1\uparrow} + D_{1\downarrow}}$$

$$TMR = \frac{R_{AP} - R_P}{R_P} = \frac{G_P - G_{AP}}{G_{AP}} = \frac{2P_1P_2}{1 - P_1P_2}$$

It works, especially in case of Al_2O_3 barriers.

Fe/MgO/Fe: Coherent tunneling

TMR (RT) MTJ conventional (Al2O3) ~ 70%

TMR (RT) MTJ Fe/MgO/Fe ~ 800% (theoretical value = **1000%**)

Giant room-temperature magnetoresistance in single-crystal Fe/MgO/Fe magnetic tunnel junctions

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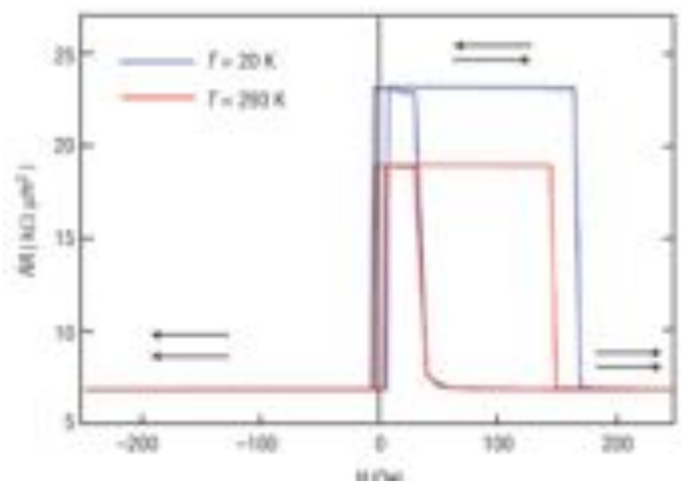
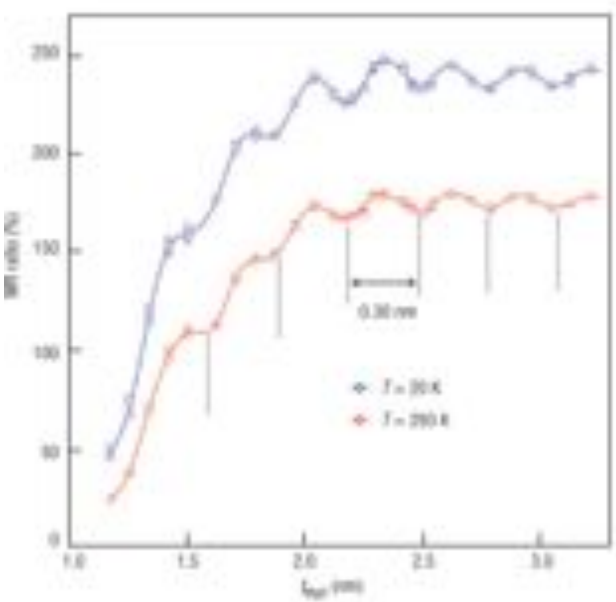
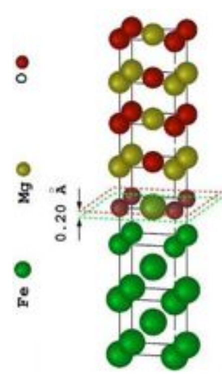
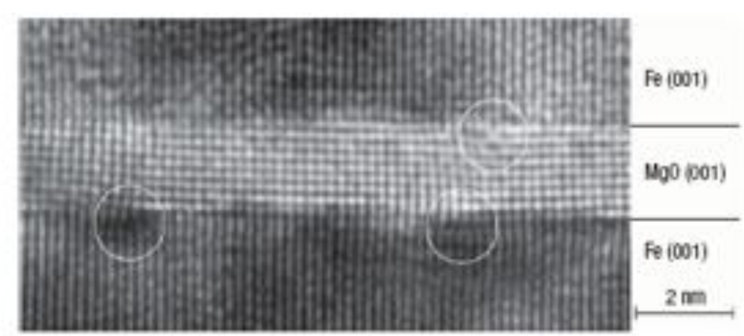


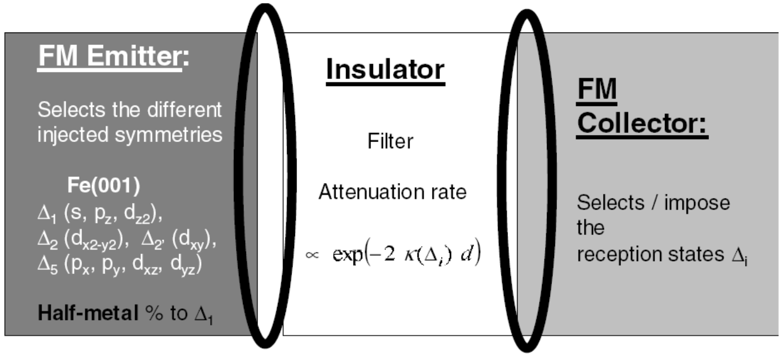






S. Yuasa et al, *Nature Materials*, ***3*** *868 (2004)*

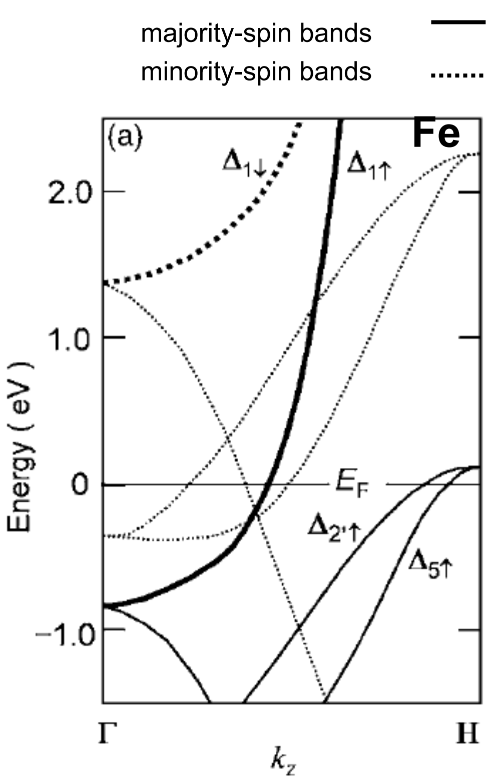
Coherent tunneling



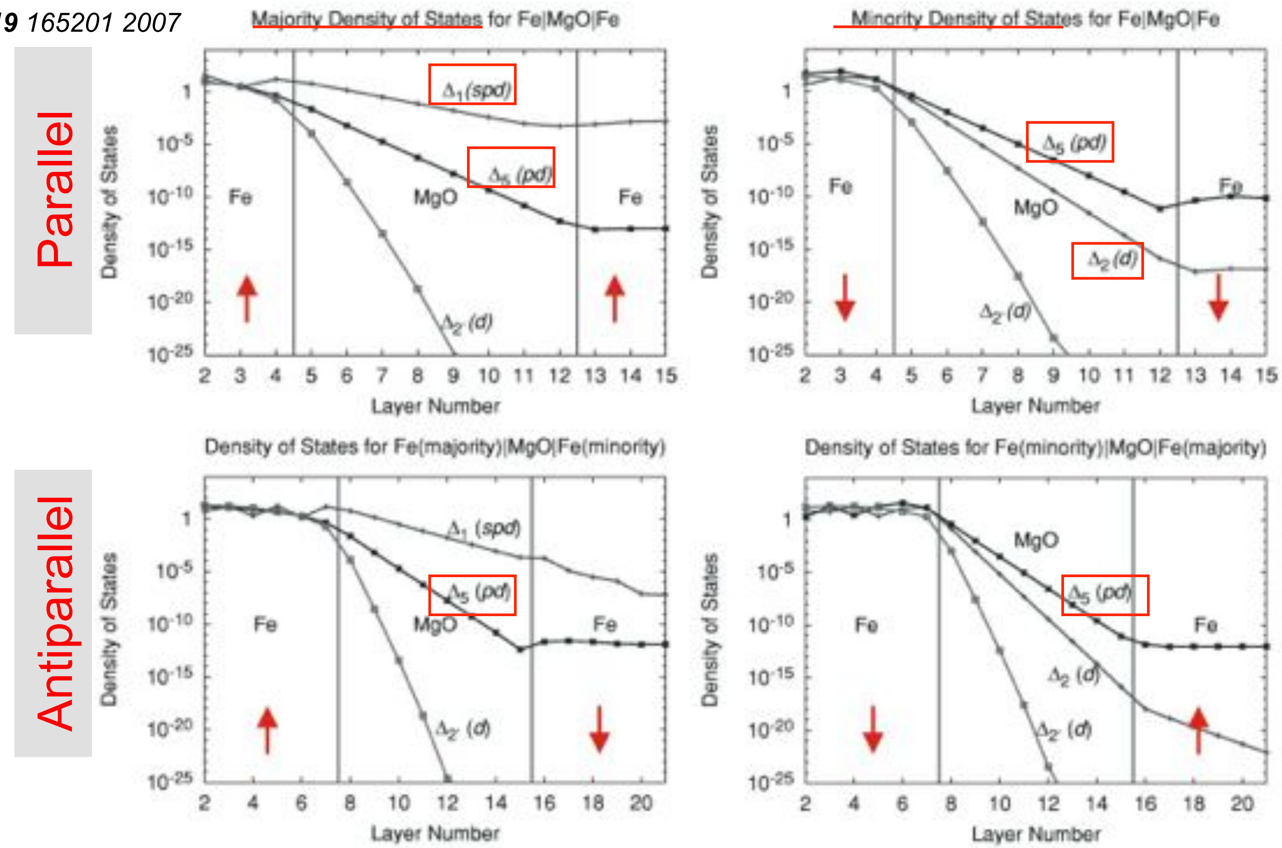
*C. Tiusan et al, J.Phys.:Cond. Matter **19** 165201 2007*

Symmetry based spin filtering

- coupling of electronic states in the collector and emitter through the MgO barrier (FM emitter, FM collector)
- different attenuation (*k*) in the barrier depending on the symmetry of states



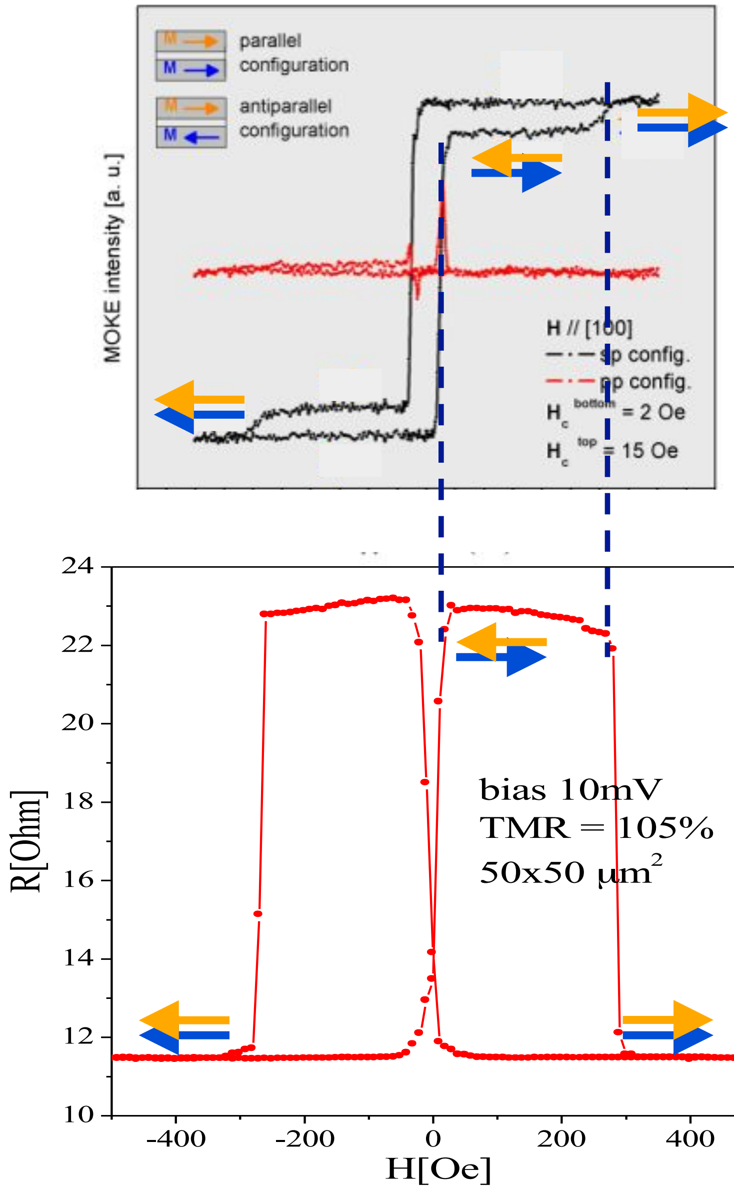
*S. Yuasa et al, APL **89** 042505 2006*



*W. H. Butler et al, PRB **63** 054416 (2001)*

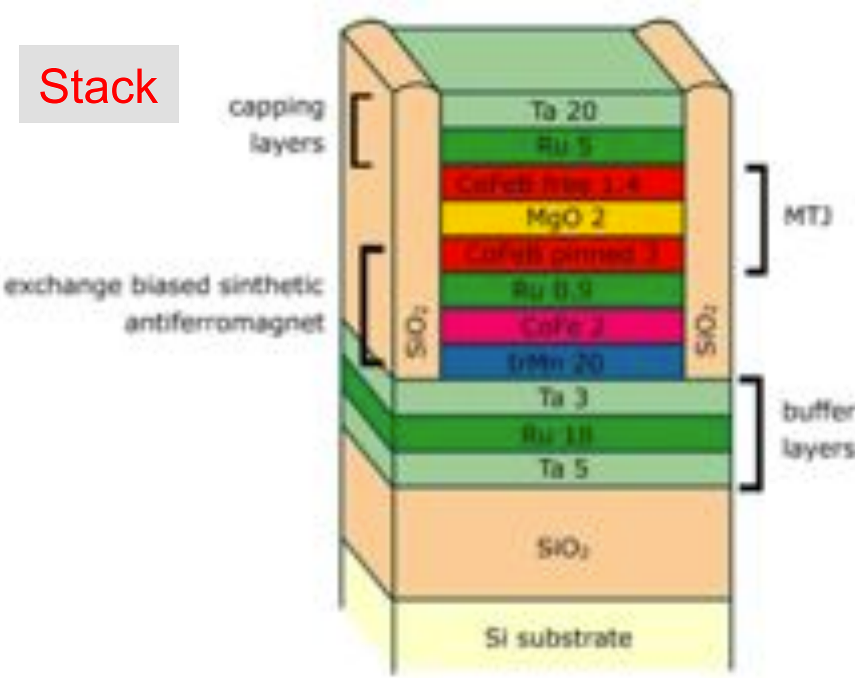
TMR measurements

Basic principle

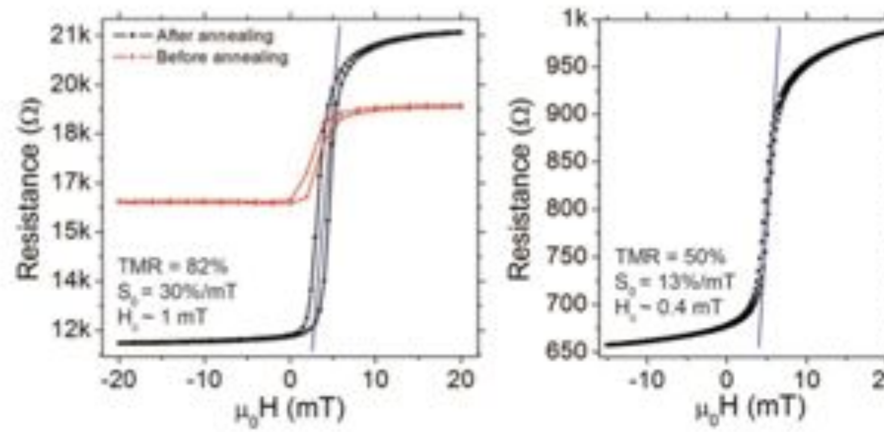


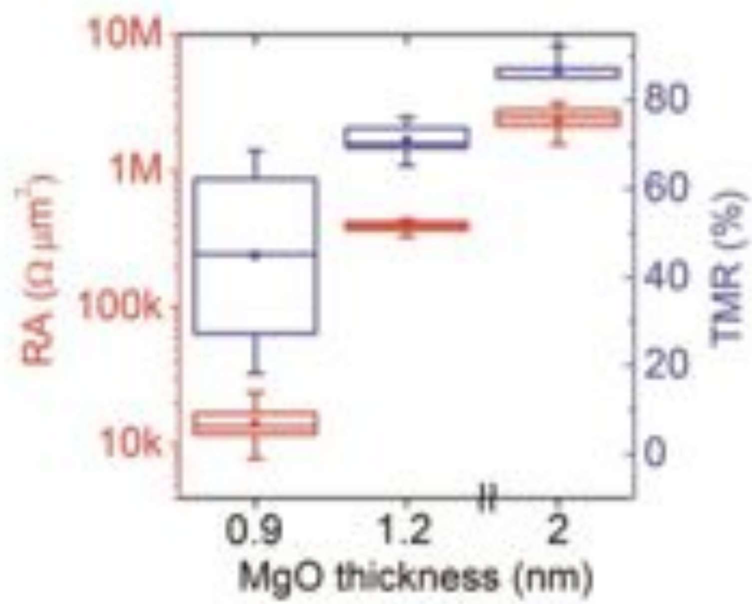
A state of the art TMR sensor

Stack

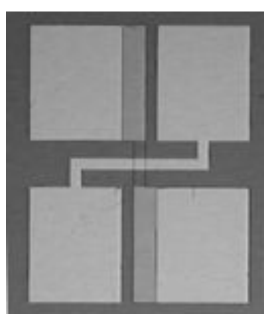


Magnetoresistive behaviour





Fabrication



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Intrinsic spin–orbit interaction in an atom

A negatively charged electron in an atom feels the electric field due to the positively charged nucleus.

$$\vec{B} = \frac{\vec{\mathcal{E}} \times \vec{v}}{c^2\sqrt{1 - v^2/c^2}},$$

Thomas (Nature in 1926) had pointed out that the Lorentz transformation that we normally use to connect the electron's rest frame to the laboratory frame is inexact. If there is a component of the electric field in a direction perpendicular to the instantaneous velocity, the electron will be accelerating perpendicular to the velocity.

$$\vec{B} = \frac{\vec{\varepsilon} \times \vec{v}}{2c^2\sqrt{1 - v^2/c^2}}$$

The relativistic Zeeman-like interaction energy is:

$$E_{rel} = -\vec{\mu}_e \cdot \vec{B}.$$

Spin

$$\vec{\mu}_e = -g_0 \mu_B \vec{S}$$

Orbital motion

$$\mu_B = \frac{e\hbar}{2m}$$

e : modulus of the electron charge

$$E_{rel} = -\left(-g_0 \mu_B \vec{S}\right) \cdot \frac{\vec{\varepsilon} \times \vec{v}}{2c^2 \sqrt{1 - v^2/c^2}} = g_0 \frac{e\hbar}{2m} \frac{\vec{\varepsilon} \times \vec{v}}{2c^2 \sqrt{1 - v^2/c^2}} \cdot \vec{S}$$

To obtain the spin-orbit hamiltonian we replace vectors **S** and **p** with the corresponding operator

$$\vec{S} = \frac{1}{2}\vec{\sigma} \qquad \vec{p} = i\hbar\nabla$$

$$H_{SO} = g_0 \frac{e\hbar}{2m} \frac{\vec{\varepsilon} \times \vec{v}}{2c^2 \sqrt{1 - v^2/c^2}} \cdot \frac{\vec{\sigma}}{2} = \frac{e\hbar}{4mc^2} \frac{\vec{\varepsilon} \times \vec{v}}{\sqrt{1 - v^2/c^2}} \cdot \vec{\sigma} = \qquad \textit{(}g_0\textit{=2 in vacuum)}$$

$$= \frac{e\hbar}{4mc^2} \frac{(-\nabla V) \times (\vec{p}/m)}{\sqrt{1 - v^2/c^2}} \cdot \vec{\sigma} \approx -\frac{e\hbar}{4m^2c^2}\left(\nabla V \times \vec{p}\right) \cdot \vec{\sigma} \qquad \textit{(if } v << c\textit{)}$$

Consider that:

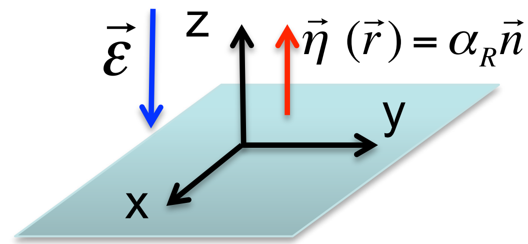
$$-e\nabla V = \frac{\partial U}{\partial r}\frac{\vec{r}}{r} \qquad \vec{r} \times \vec{p} = \hbar\vec{L}$$

$$H_{SO} = \frac{1}{2m^2c^2r}\frac{\partial U}{\partial r}\vec{L} \cdot \vec{S}$$

Rashba in 2DEG

The Rashba Hamiltonian in absence of a magnetic field is:

$$H_R = \vec{\eta}_R(\vec{r})\cdot(\vec{\sigma}\times\vec{p}) = \frac{\eta}{\hbar}\hat{z}\cdot(\vec{\sigma}\times\vec{p}) = \eta\hat{z}\cdot\left(\vec{\sigma}\times\vec{k}\right)$$



If the motion is confined in the x,y plane:

$$\vec{k} = k_x\vec{i} + k_y\vec{j}$$

$$\vec{\sigma}\times\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{n} \\ \sigma_x & \sigma_y & \sigma_z \\ k_x & k_y & 0 \end{vmatrix} = \vec{i}\,(-\sigma_z k_y) + \vec{j}\,(\sigma_z k_x) + \vec{n}(\sigma_x k_y - \sigma_y k_x)$$

$$\eta = \alpha_R$$

$$H_R = \alpha_R(\sigma_x k_y - \sigma_y k_x) = \alpha_R\begin{bmatrix} 0 & k_y + ik_x \\ k_y - ik_x & 0 \end{bmatrix} \qquad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

In the “variable separation approximation”, assuming there is no dependence on z and neglecting energy contributions arising from quantum confinement in z:

$$\Psi_{2D} = e^{ik_x x}e^{ik_y y}\begin{bmatrix} a \\ b \end{bmatrix} = e^{ik_x x}e^{ik_y y}\chi$$

$$H = \frac{\hbar^2 k^2}{2m} + H_R = \begin{bmatrix} \frac{\hbar^2 k^2}{2m} & \alpha_R \left(k_y + ik_x\right) \\ \alpha_R \left(k_y - ik_x\right) & \frac{\hbar^2 k^2}{2m} \end{bmatrix}$$

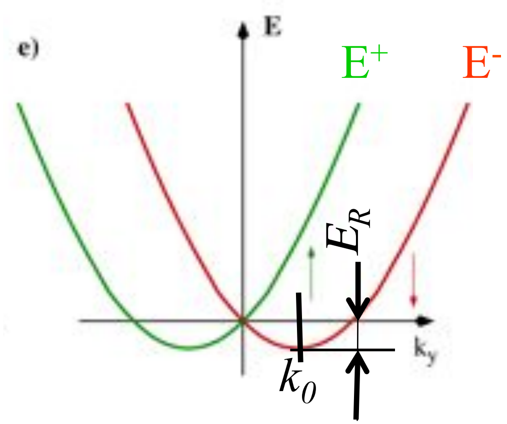
Eigenvalues

$$\det \begin{bmatrix} \frac{\hbar^2 k^2}{2m} - E & \alpha_R \left(k_y + ik_x\right) \\ \alpha_R \left(k_y - ik_x\right) & \frac{\hbar^2 k^2}{2m} - E \end{bmatrix} = 0$$

$$\left(\frac{\hbar^2 k^2}{2m} - E\right)^2 - \alpha_R^2 \left(k_y^2 + k_x^2\right) = 0$$

$$\frac{\hbar^2 k^2}{2m} - E^{\pm} = \pm \alpha_R |k|$$

$$E^{\pm} = \pm \alpha_R |k| + \frac{\hbar^2 k^2}{2m}$$



Rashba parameters

$$E^{-} = -\alpha_R |k_y| + \frac{\hbar^2 k_y^2}{2m} \qquad \text{for } k_y>0$$

The minimum is found for k_0 such as

$$\frac{dE^-}{dk_y} = -\alpha_R + \frac{\hbar^2 k_0}{m} = 0$$

$$\boxed{k_0 = \frac{m\alpha_R}{\hbar^2}}$$

The k-splitting is $2k_0$!

The Rashba energy $E_R=|E^-(k_0)|$

$$\boxed{E_R = \left|E^-(k_0)\right| = \frac{m\alpha_R^2}{2\hbar^2}}$$

Eigenvectors

$$H = \frac{\hbar^2 k^2}{2m} + H_R = \begin{bmatrix} \dfrac{\hbar^2 k^2}{2m} & \alpha_R\left(k_y + ik_x\right) \\ \alpha_R\left(k_y - ik_x\right) & \dfrac{\hbar^2 k^2}{2m} \end{bmatrix}$$

$$\left(H - E^{\pm} I\right)\chi^{\pm} = 0$$

For E^+

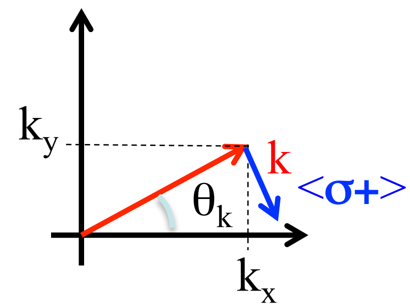
$$\begin{bmatrix} -\alpha_R |k| & \alpha_R\left(k_y + ik_x\right) \\ \alpha_R\left(k_y - ik_x\right) & -\alpha_R |k| \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$$\begin{cases} -|k| a + \left(k_y + ik_x\right) b = 0 \\ \left(k_y - ik_x\right) a - |k| b = 0 \end{cases}$$

$$a = b\frac{k_y + ik_x}{|k|}$$

$$a = b\frac{|k|}{k_y - ik_x} = b\frac{k_y + ik_x}{|k|}$$

$$\chi^+ = \frac{1}{\sqrt{2}} \begin{bmatrix} \dfrac{k_y + ik_x}{|k|} \\ 1 \end{bmatrix}$$



Exercise: Demonstrate that **S** and **k** are perpendicular

$$\left\langle \sigma_x \right\rangle = \left\langle \chi_+ \middle| \sigma_x \middle| \chi_+ \right\rangle = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \dfrac{k_y + ik_x}{k} \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} 1 \\ \dfrac{k_y + ik_x}{k} \end{bmatrix} = \frac{k_y}{k} = \sin\vartheta_k$$

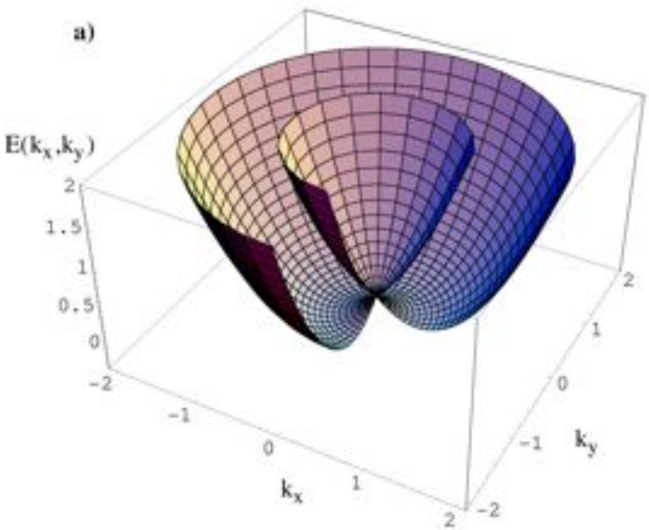
$$\left\langle \sigma_y \right\rangle = \left\langle \chi_+ \middle| \sigma_y \middle| \chi_+ \right\rangle = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} \dfrac{k_y + ik_x}{k} \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} -i \\ i\dfrac{k_y + ik_x}{k} \end{bmatrix} = -\frac{k_x}{k} = -\cos\vartheta_k$$

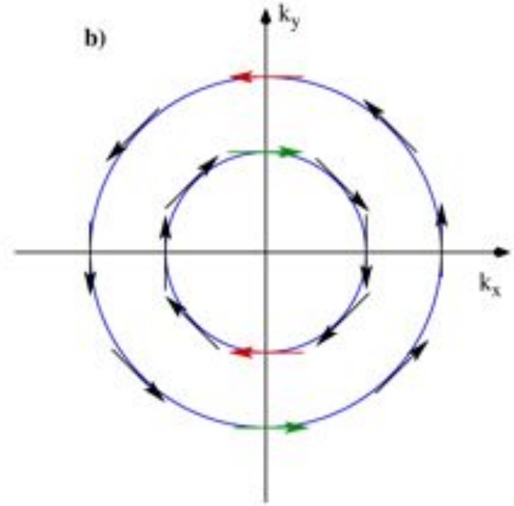
$$\left\langle \sigma_z \right\rangle = \left\langle \chi_+ \middle| \sigma z \middle| \chi_+ \right\rangle = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \dfrac{k_y + ik_x}{k} \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \dfrac{k_y - ik_x}{k} & 1 \end{bmatrix}\begin{bmatrix} \dfrac{k_y + ik_x}{k} \\ -1 \end{bmatrix} = 0$$

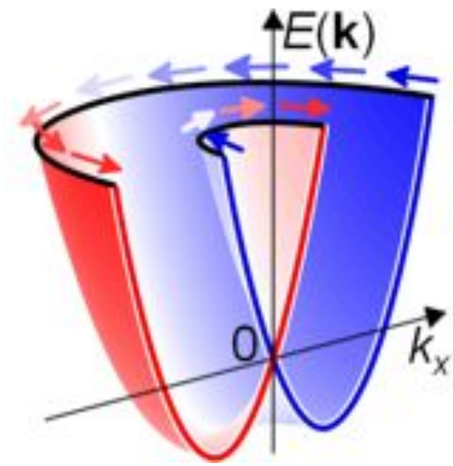
$$\left\langle \vec{\sigma} \right\rangle \cdot \vec{k} = \frac{k_y}{k} k_x + \frac{k_x}{k}\left(-k_x\right) = 0$$

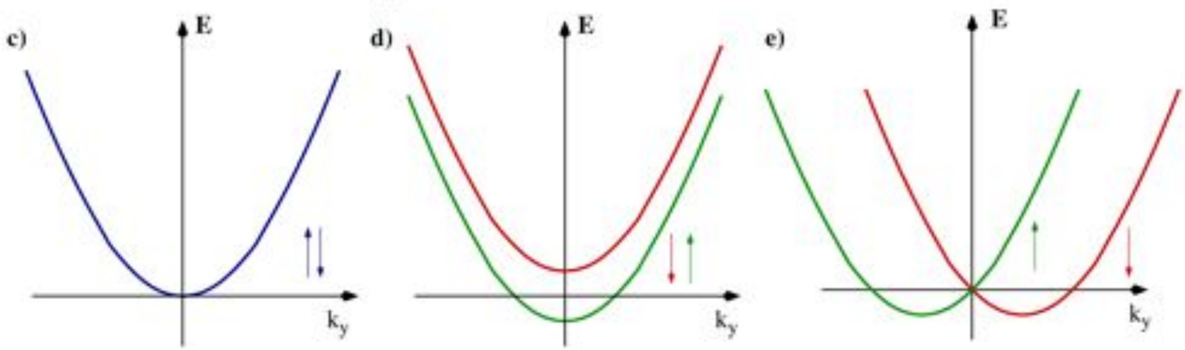
This is the sense of circulation in the high energy band.
(inner circle in the isoenergy cut)

Spin texture in Rashba bands









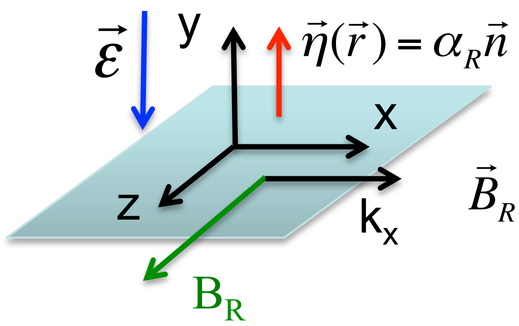
unperturbed

Zeeman splitting

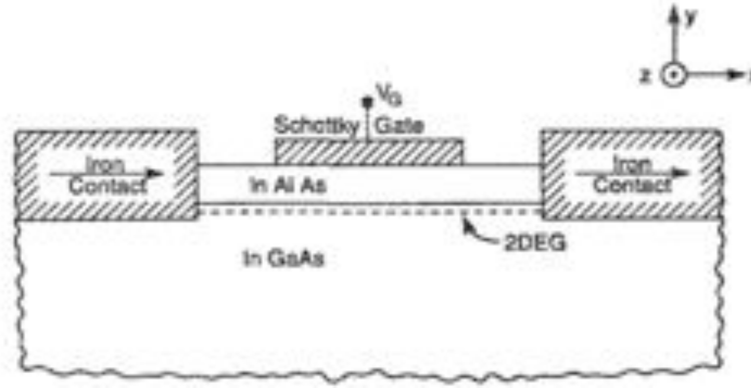
Rashba splitting

Spin FET (Datta & Das 1990)

For positive V_G, electrons moving along x (k_x)



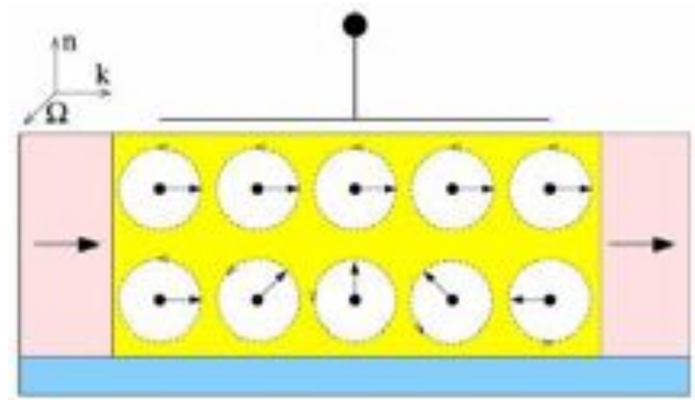
$$\vec{B}_R = -\frac{2\alpha_R}{g\mu_B}\hat{y}\times\hat{k}$$



Particle viewpoint

A spin injected at the source with the spin along x will precess around B_R

$$\frac{d\mathbf{S}}{dt} = \Omega\times\mathbf{S} = \frac{g\mu_B\mathbf{B}_R}{\hbar}\times\mathbf{S}$$



We use the spin-independent Rashba field

$$B_{Rashba} = \left[\frac{2m^* a_{46}}{g\mu_B\hbar}\mathcal{E}_y v_x\right]\hat{z}$$

$$\Omega = \frac{d\phi}{dt} = \frac{g\mu_B B_{Rashba}}{\hbar} = \frac{2a_{46}m^*}{\hbar^2}\mathcal{E}_y v_x$$

$$\frac{d\phi}{dx} = \frac{d\phi}{dt}\frac{dt}{dx} = \frac{d\phi}{dt}\frac{1}{v_x} = \frac{2a_{46}m^*}{\hbar^2}\mathcal{E}_y$$

$$\Phi = \left[\frac{2a_{46}m^*}{\hbar^2}\mathcal{E}_y\right]L$$

$(2\nu+1)\pi$, OFF

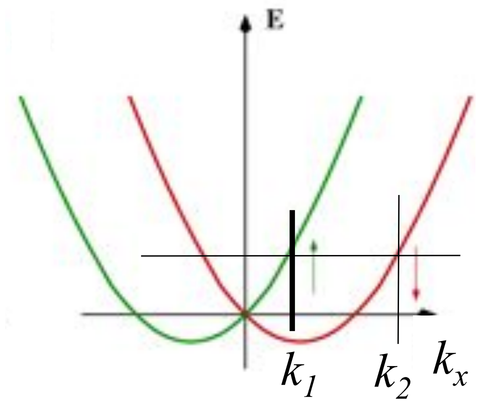
2π, ON

Independent on v, stable againts collisions

Spinors viewpoint

Electrons at the source are injected with the spin along x and must travel along x:

$$\Psi_{source} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}$$

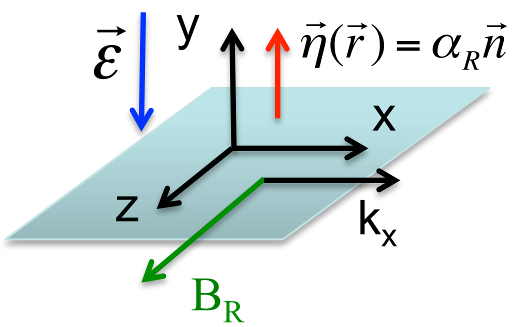


The eigenstates of the Rashba Hamiltonian are:

$\Psi^+ = \begin{bmatrix}1\\0\end{bmatrix}$ *upper band, a spin injected at E* has K_1*

$\Psi^- = \begin{bmatrix}0\\1\end{bmatrix}$ *lower band, a spin injected at E* has K_2*

$$H_R = \eta\hat{y}\cdot\left(\vec{\sigma}\times\vec{k}\right) = \eta\hat{y}\cdot\begin{vmatrix}\hat{x} & \hat{y} & \hat{k}\\ \sigma_x & \sigma_y & \sigma_z\\ k_x & 0 & 0\end{vmatrix} = \eta k_x \sigma_z$$



Whose eigenvalues are $\pm\eta k_x$ for $\Psi^{\pm}$

The total energy eigenvalues are:

$$E_+^* = \frac{\hbar^2 k_1^2}{2m} + \eta k_1 \quad \text{for} \quad \Psi^+$$

$$E_-^* = \frac{\hbar^2 k_2^2}{2m} - \eta k_2 \quad \text{for} \quad \Psi^-$$

$$E_+^* = E_-^*$$

$$k_2 - k_1 = \frac{2m\eta}{\hbar^2}$$

$$\Phi = \left(k_2 - k_1\right)L = \frac{2m\eta}{\hbar^2}L$$

As in the previous slide because $\eta = a_{46}\varepsilon$

At the drain:

$$\Psi_{drain} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix}e^{ik_1L} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}e^{ik_2L} = \frac{1}{\sqrt{2}}\begin{bmatrix}e^{ik_1L}\\e^{ik_2L}\end{bmatrix}$$

Electrons in the drain have spinors $(1/\sqrt{2})[1\ 1]^{\dagger}$.

The transmission is thus: $t(E) = \frac{1}{\sqrt{2}}[1\ 1]\frac{1}{\sqrt{2}}\begin{bmatrix}e^{ik_1L}\\e^{ik_2L}\end{bmatrix} = \frac{1}{2}\left(e^{ik_1L} + e^{ik_2L}\right)$

And the transmission probability:

$$T(E) = |t(E)|^2 = \frac{1}{4}\left|1 + e^{i(k_2-k_1)L}\right|^2 = cos^2\left(\frac{\Phi}{2}\right) \qquad \Phi = \left(k_2 - k_1\right)L = \frac{2m\eta}{\hbar^2}L$$

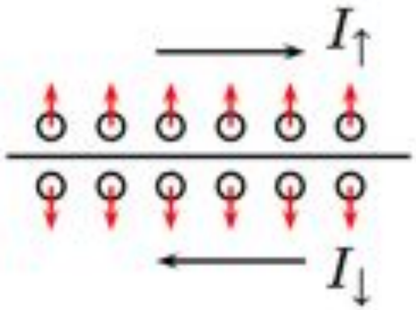
$$\Psi_{drain} = \frac{1}{\sqrt{2}}e^{ik_1L}\begin{bmatrix}1\\e^{-i(k_2-k_1)L}\end{bmatrix}$$

Φ=(2n+1)π LOW transmission; OFF state

Pure spin currents (PSC)

Key concept – Spin current:

- Spin transport
- Spin-based information exchange
- More general than the "spin polarized current"



Intuitive definition of spin current:

$$I_s = I_\uparrow - I_\downarrow$$

Pure spin current:

$$I_\uparrow = -I_\downarrow$$

$$I_c = I_\uparrow + I_\downarrow = 0$$

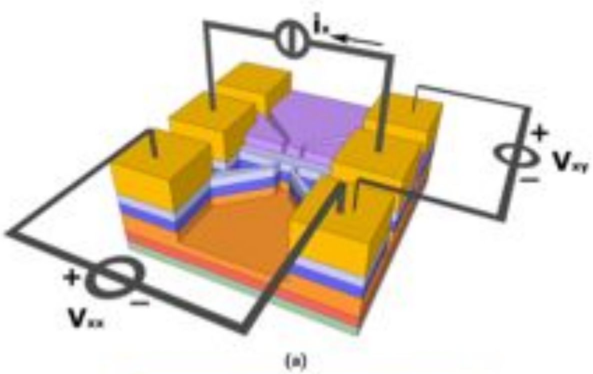
$$I_s = I_\uparrow - I_\downarrow = 2I_\uparrow$$

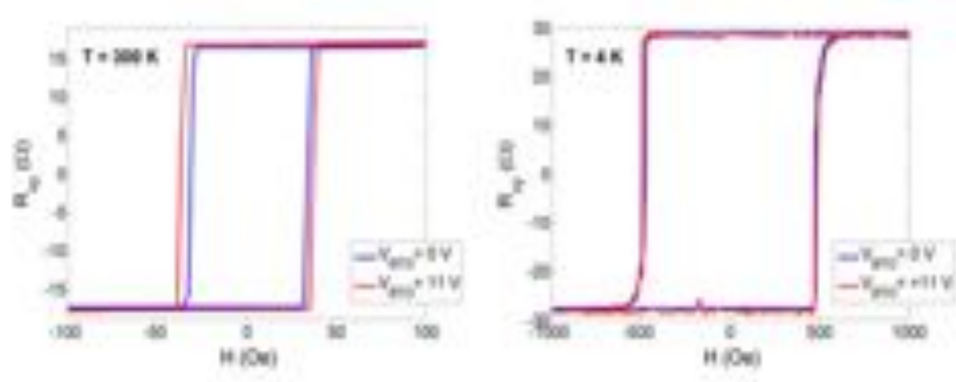
Anomalous Hall effect

Because of spin-dependent band structure or spin-dependent scattering events due to spin-orbit coupling, electrons whose spins are polarized in the +z-direction, are scattered to one edge of the sample and electrons whose spins are polarized in the −z-direction are scattered to the other edge.
In a ferromagnetic sample with non-zero magnetization there is an unbalance of spin up and down electrons: thus a charge and spin unbalance is created at the two edges of the samples in the y direction.
We expect both charge currents and spin currents!

$$\rho_H = R_0 B + R_S M$$

R_0: ordinary Hall coefficient
R_S: anomalous Hall coefficient

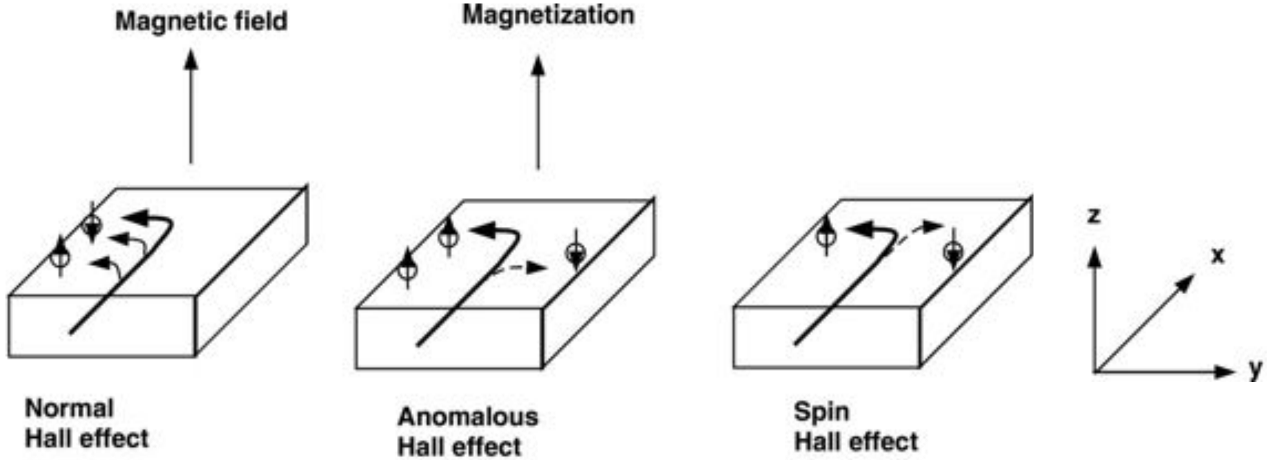




(a) (b)

CoFeB with perpendicular magnetic anisotropy on $BaTiO_3$ (unpublished)

Spin-Hall effect (SHE)



Extrinsic SHE

VOLUME 83, NUMBER 9 PHYSICAL REVIEW LETTERS 30 AUGUST 1999

Spin Hall Effect

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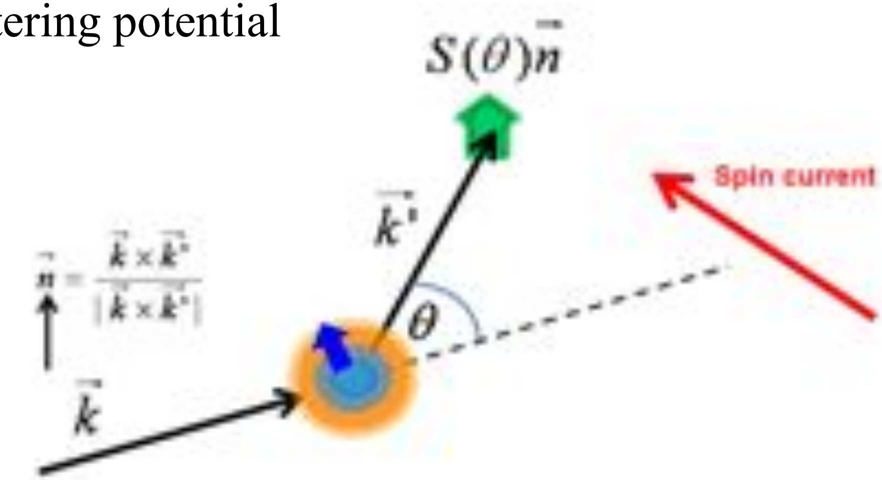
(Received 24 February 1999)

If a current flows in the x-direction within a paramagnetic semiconductor, due to spin dependent scattering phenomena (e.g. SO) as in the case of anomalous Hall effect, spin up electrons are deflected to the left and spin down to the right.
As the sample is not ferromagnetic, there is not net unbalance between spin up and down electrons. Thus there is no charge current but a pure spin current (PSC).

For an unpolarized electron beam incident on a SO scattering potential of the form:

$$V = V_c(r) + V_s(r)\vec{\sigma} \cdot \vec{L}$$

σ, L: electron spin and orbital momentum
V_s: SO scattering potential

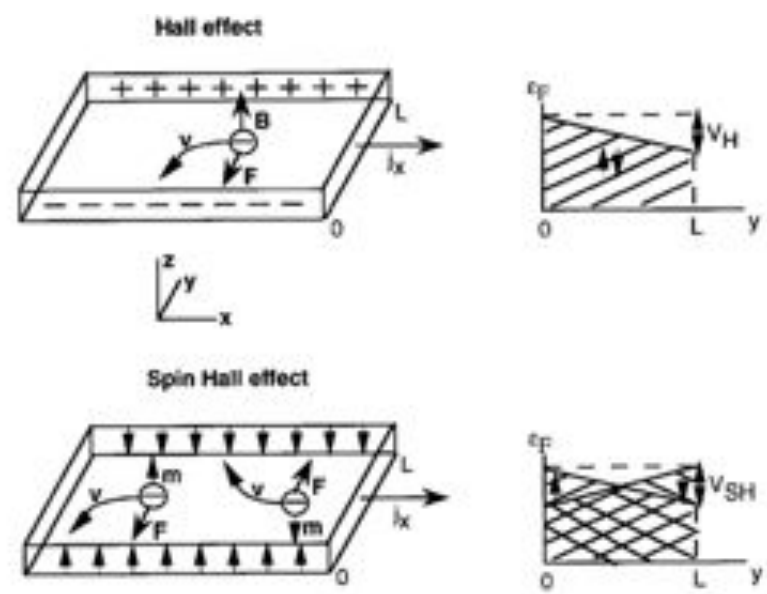


The scattered beam will have a polarization vector:

$$\vec{P}_f = \frac{fg^* + f^*g}{|f|^2 + |g|^2}\hat{n},$$

Where f (g) is the spin independent (dependent) part of the scattering amplitude.

As **n** is opposite for electrons scattered on the left and on the right, there's a left-right asymmetry to the spin polarization of the scattered beam.



There's is a fundamental difference with respect to ordinary Hall effect:
the Fermi levels for each spin electrons will also be different on both sides of the sample, but the difference will be of opposite sign for both spins. No net voltage difference.

No charge current, but pure spin current (PSC)!

Estimate the associate spin-voltage and spin current

The scattering processes are the same of the anomalous Hall effect. Imagine now that we have only spin up electrons with associated "magnetization":

$$M_\uparrow = n_\uparrow \mu_B$$

$$V_H^\uparrow = j_x^\uparrow L \cdot R_S M_\uparrow = j_x^\uparrow L \cdot R_S n_\uparrow \mu_B$$

Spin down electrons moving along x will produce an opposite spin voltage, so that the total spin voltage is:

$$V_H = 2 j_x^\uparrow L \cdot R_S M_\uparrow = 2 j_x^\uparrow L \cdot R_S n_\uparrow \mu_B = j_x L \cdot R_S \frac{n}{2} \mu_B$$

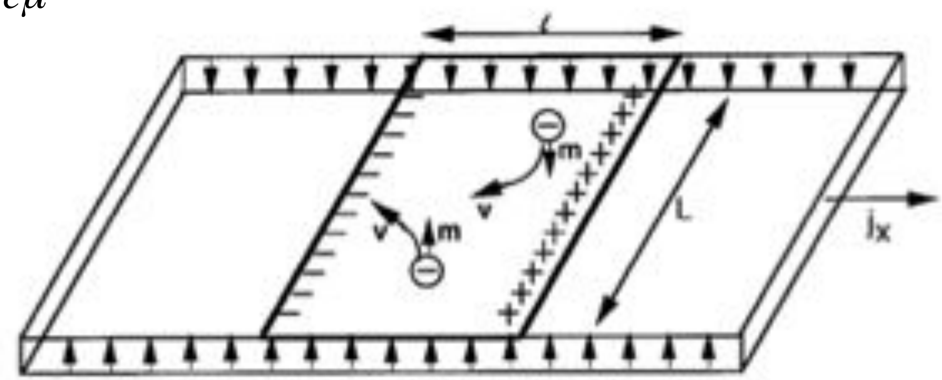
Assuming that the resistivity for the spin current is the same as that for the charge current we have for the spin current:

$$J_s = \frac{V_{SH}}{L}\sigma = j_x L \cdot R_S \frac{n}{2}\mu_B \frac{ne\mu}{L} = j_x R_S \frac{n^2}{2} e\mu$$

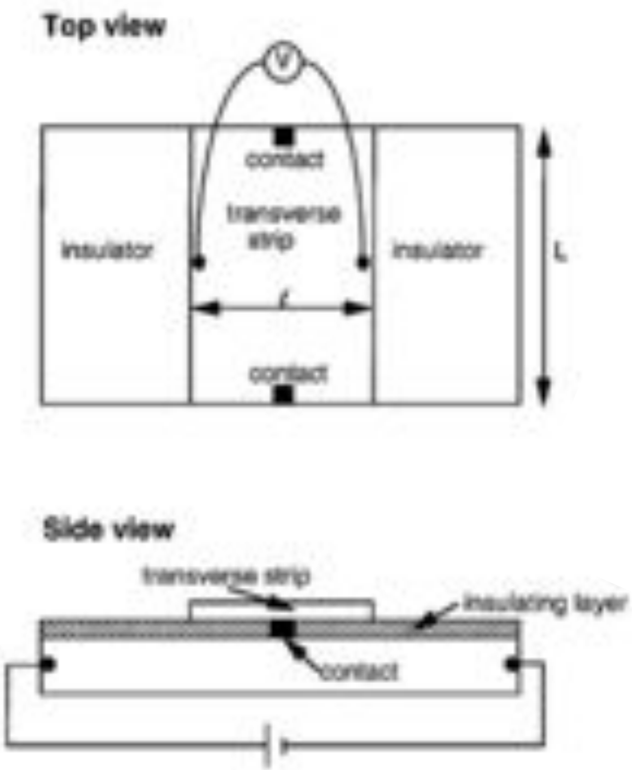
The spin-Hall angle is defined as:

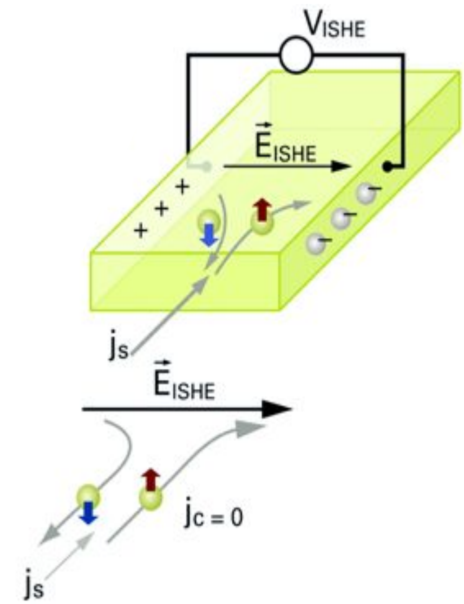
$$\vartheta_{SH} = \frac{J_{spin}}{J_{charge}} = R_S \frac{n^2}{2} e\mu$$

$\vec{E} \cdot \vec{J}_S = 0$ Dissipationless!!



Inverse spin-Hall effect





$$\vec{\mathcal{E}}_{ISH} = \theta_{SH}\rho\left[\vec{J}_{\sigma} \times \vec{\sigma}\right]$$

where σ is the spin polarization of the PSC

A PSC will flow in the top strip, but the two spin up and spin down charge currents are now antiparallel so that the spin-hall voltages add up to produce a net voltage.

$$V_{ISHE} = 2J_S l \cdot R_S \frac{n}{2}\mu_B = 2l \cdot R_S^2 \left(\frac{n}{2}\mu_B\right)^2 j_x$$