



Advanced magnetic materials and devices for biomedical applications

Aula Cannizzaro - Turin, 21-25 May 2018

Magnetism in reduced dimensions

Giovanni Carlotti

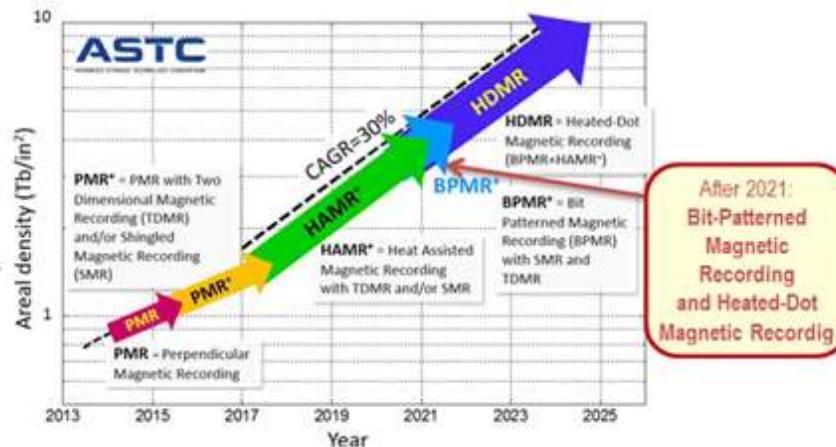
Dipartimento di Fisica e Geologia, Università di Perugia, Italy

- Time- and space-scale in the «middle-earth» of magnetism
- An outlook on devices and applications
- Theoretical approach: Landau free energy functional
- Micromagnetism
- Dynamics and spin waves
- Conclusions

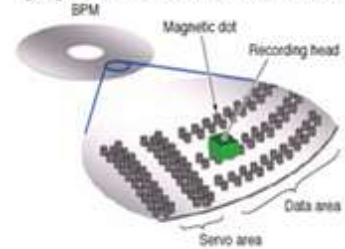
A gallery of nanomagnetic devices

(a) Magnetic Recording Roadmap

(From www.idema.org/ International Disk Drive Equipment and Material Association, that includes the world leading manufacturers, with an income of \$35 billions. Seagate and Western Digital are the main two companies)

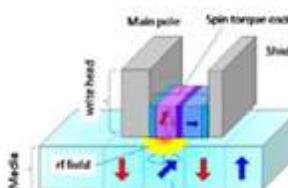


(b) Bit-patterned Hard Disk



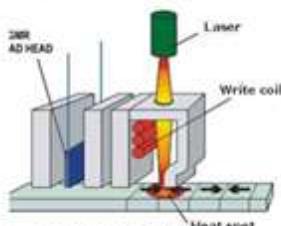
R. Griffiths et al. J. Phys. D Appl. Phys. 46, 503001 (2013)

(c) Microwave-assisted magn. recording (MAMR)

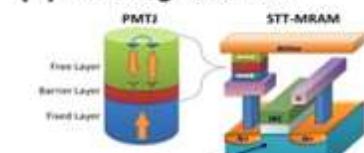


S. Okamoto et al. J. Phys. D Appl. Phys. 48, 353001 (2015)

(d) Heat-assisted magn. recording (HAMR)

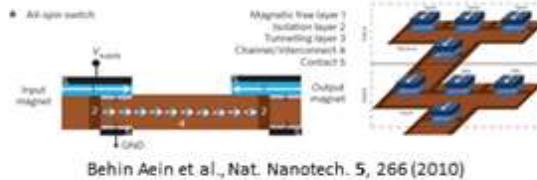


(e) STT Magnetic RAM



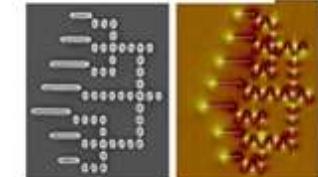
A. Kent, Nat. Nanotech. 10, 187 (2015)

(f) All-Spin Logic Devices (ASLD)



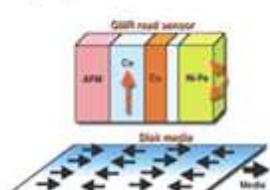
Behin Aein et al., Nat. Nanotech. 5, 266 (2010)

(g) Nano-Magnetic Logic (NML)



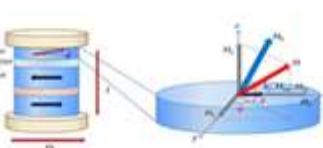
M. T. Niemier et al., J. of Physics: Cond. Mat. 24, 493202 (2011)

(h) GMR Read Head



S. Thompson, J. Phys. D: Appl. Phys. 41, 093001 (2008)

(i) Spin Torque Oscillator (STO)



J.-V. Kim, Solid State Phys. 63, 217 (2012)

The next wave

Spin waves look poised to make a splash in data processing.

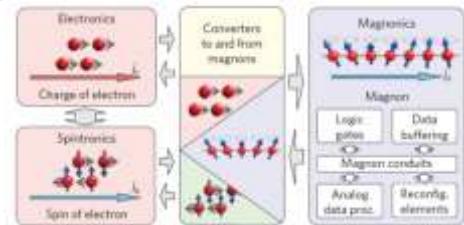
nature
physics

REVIEW ARTICLE

PUBLISHED ONLINE: 2 JUNE 2015 | DOI: 10.1038/NPHYS3347

Magnon spintronics

A. V. Chumak*, V. I. Vasyuchka, A. A. Serga and B. Hillebrands



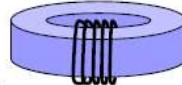
Length scale

- Macroscopic (0.1mm – ...)

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J}_0$$

- Transformers, Electrical Machines
- Power electronics
- Permanent magnets

$$\mathbf{B} = \mathcal{F}(\mathbf{H})$$



Nonlinear and possibly hysteretic

- Mesoscopic (1nm – 10μm)

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J}_0$$

- Magnetic storage technologies
- Spintronics

$$\min_{\mathbf{M}} G(\mathbf{M})$$



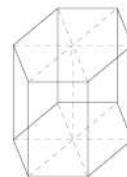
Nonconvex free energy functional

- Atomic (... – 1nm)

- Sub-nanoscale technologies
- Physical parameters

$$H = - \sum_{N(i)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_i \mu_i (\mathbf{H}_{ai} + \mathbf{H}_{mi}) \cdot \mathbf{S}_i$$

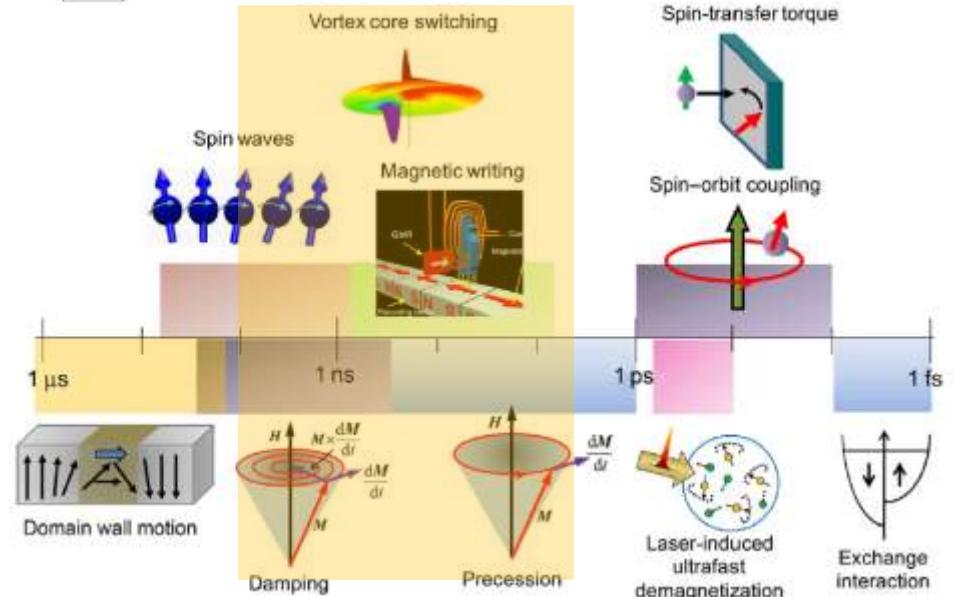
$$i\hbar \frac{d\mathbf{S}_i}{dt} = [H, \mathbf{S}_i]$$



... ideal for
ICT application

**Microwave frequencies
at micro- and nano-metric
scale.....**

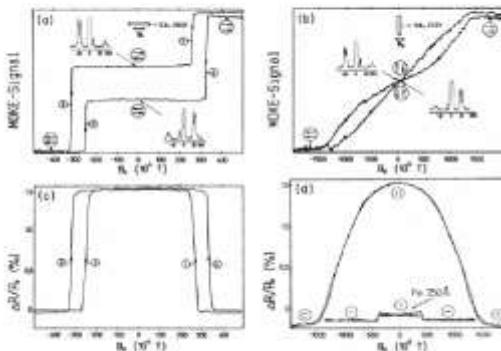
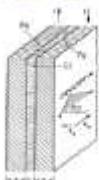
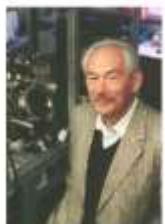
Time scale



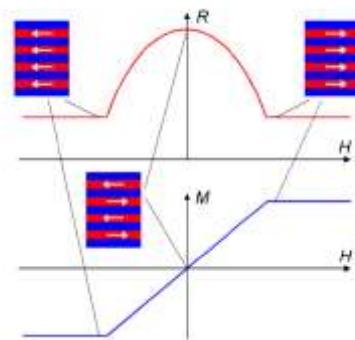
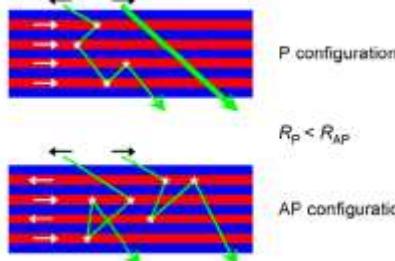
1988: Discovery of GMR – 2007: Nobel Prize in Physics

GMR in Fe/Cr/Fe trilayers

Dead last
April 7



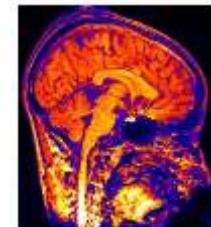
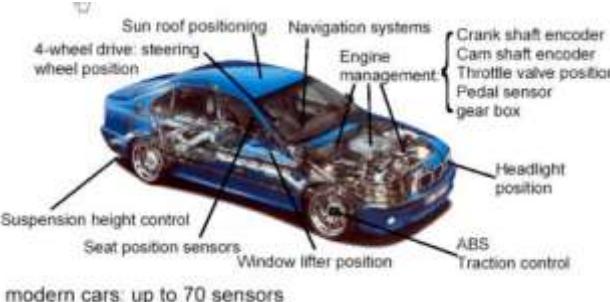
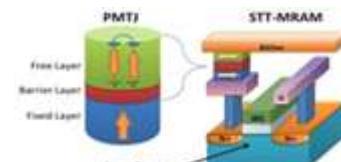
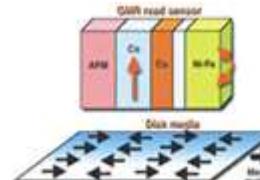
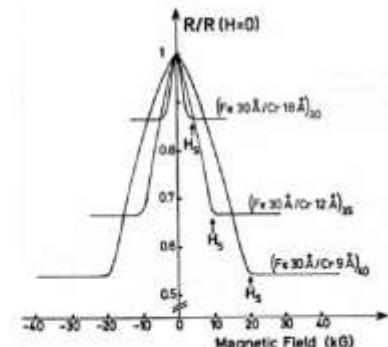
G. Binasch P. Grünberg, F. Saurenbach, W. Zinn Phys. Rev. B39, 4282 (1989)



GMR in Fe/Cr superlattices



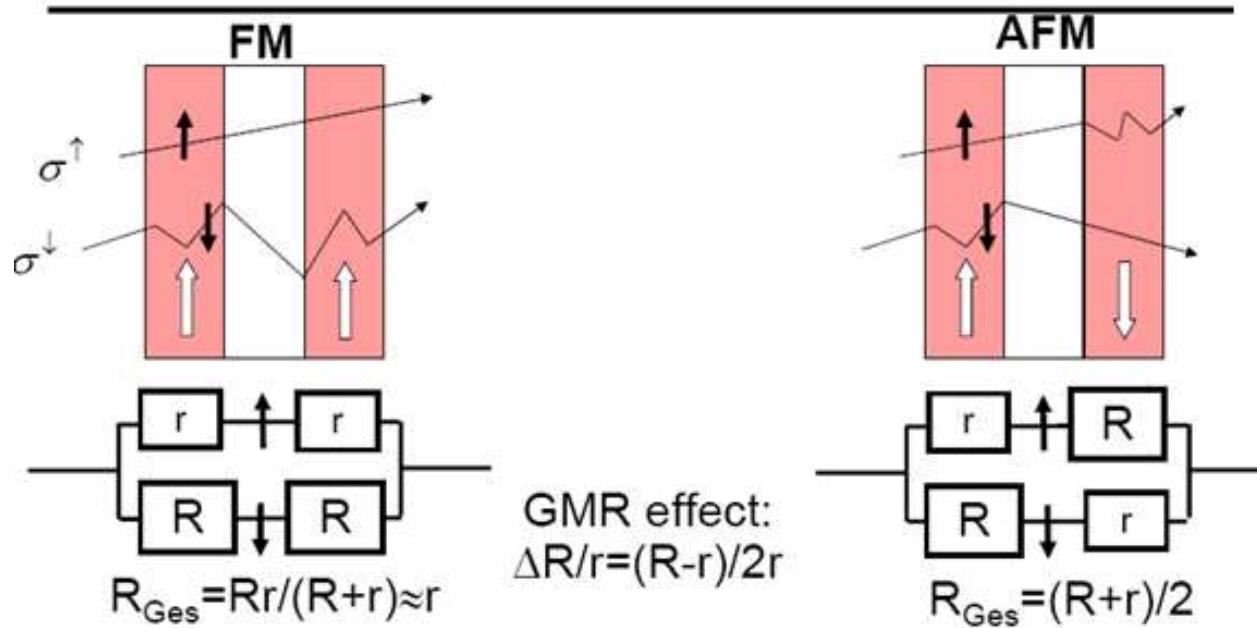
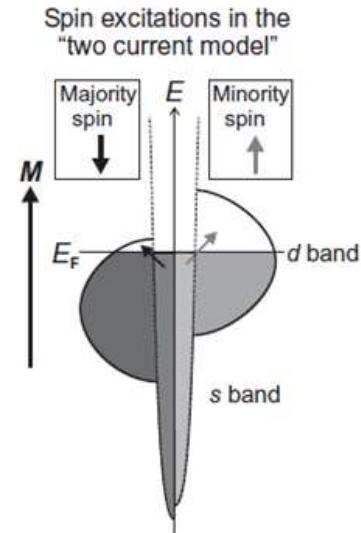
A. Fert et al.
PRB 61, 2472 (1988)



Two current model for transition metals

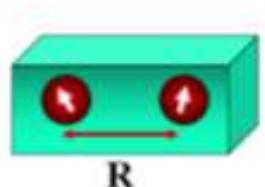
(Mott, 1936 – Fert, 1968)

- Conductivity results from the sum of the conductivities of majority and minority electrons
- Conduction is mostly due to s-electrons due to their lower effective mass (higher conductivity) if compared to d-electrons
- The main scattering mechanism is the scattering of s-electrons in empty d states, whose probability is very different for the two families of spin



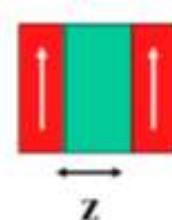
RKKY
stands for
*Ruderman–
Kittel–
Kasuya–
Yosida*

RKKY interaction in bulk materials...

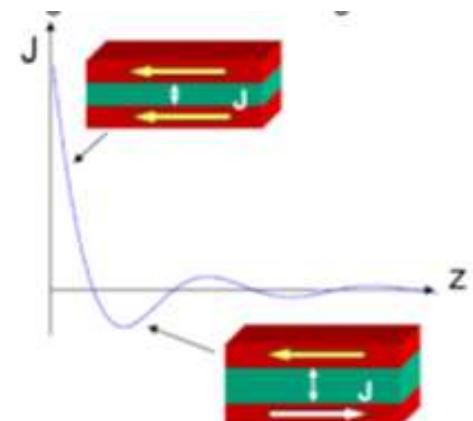


$$J_{RKKY}(R) \propto \frac{\cos(2k_F R)}{(2k_F R)^3} \propto \frac{1}{R^3}$$

... and in layered systems



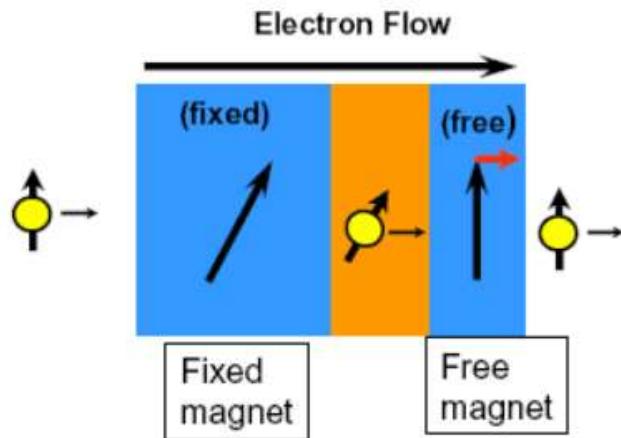
$$J_{RKKY}(R) \sim \frac{1}{z^2}$$



There is more than GMR/TMR...

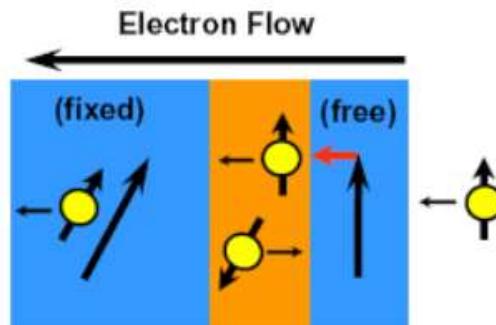
The «dual» effect Spin Transfer Torque (STT)

(predicted independently in 1996 by J. Slonczewski and L. Berger)



Fixed layer is larger or otherwise pinned so that spin transfer from the current does not excite it

Right-going electrons exert torque on free layer favoring parallel alignment



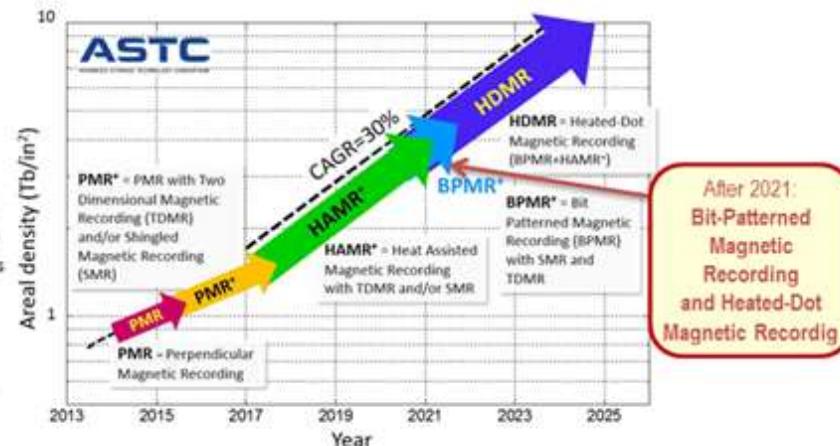
Left-going electrons are partially **reflected** back by fixed layer

Reflected electrons exert **opposite torque** on free layer

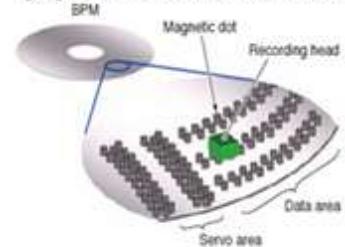
=> anti-parallel alignment is favored

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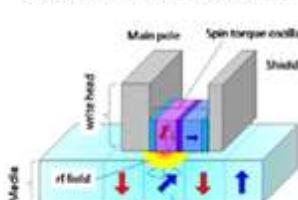


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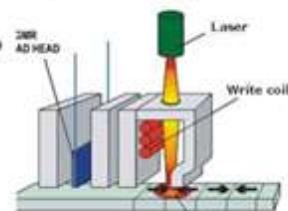
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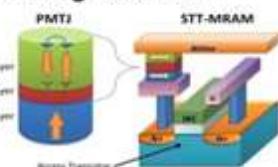
(d) Heat-assisted magn. recording (HAMR)



Again: nanomagnetic devices in the «middle earth»...

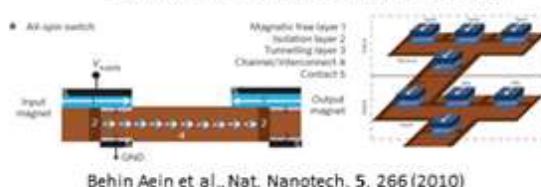
...looking for
predictive
theoretical
description

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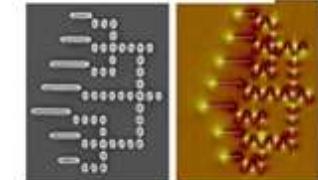
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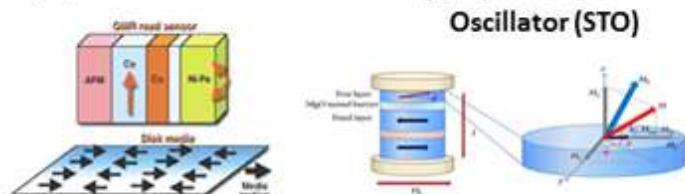
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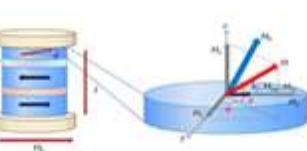
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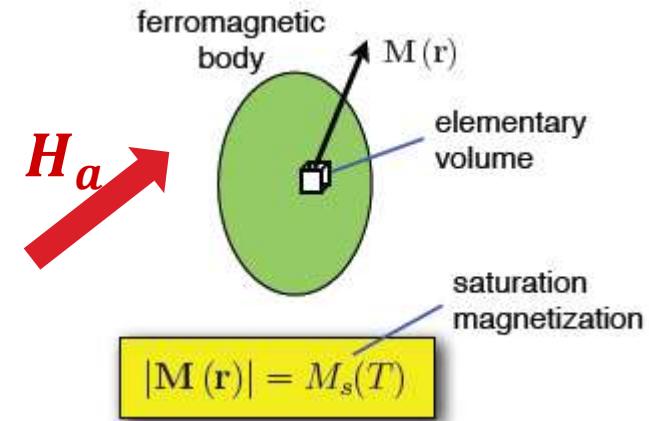
(i) Spin Torque Oscillator (STO)



J.-V. Kim, Solid State Phys. 63, 217 (2012)

Energetic approach

$$\mathbf{m}(\mathbf{r}) = \frac{\mathbf{M}(\mathbf{r})}{M_s} \quad , \quad |\mathbf{m}| = 1$$



- Continuum approximation
- The applied field H_a is known
- The dimensions of the body are lower than the wavelength of em waves, so the Magnetostatic approximation of Maxwell's equation is possible

Landau Free Energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) \, dV$$

body material shape applied field

Exchange energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) \, dV$$

$$\mathcal{H} = - \sum_{} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

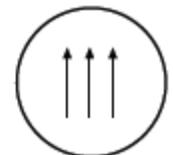
$$G_{\text{ex}} = \int_{\Omega} A [(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2] = \int_{\Omega} A (\nabla \mathbf{m})^2 \, dV$$

$$A = \frac{1}{6} nJS^2 \sum \Delta r_j^2 \quad \text{is the stiffness constant } \approx 10^{-11} \text{ J/m}$$

exchange



uniform magn.



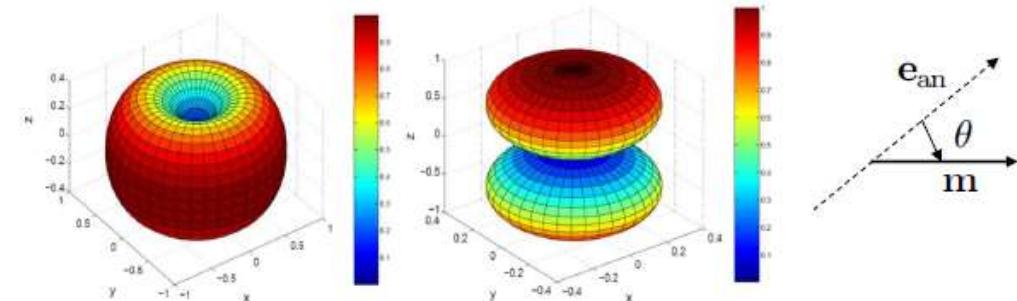
Anisotropy energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) \, dV$$

$$G_{an} = \int_{\Omega} f_{an}(\mathbf{m}) \, dV$$

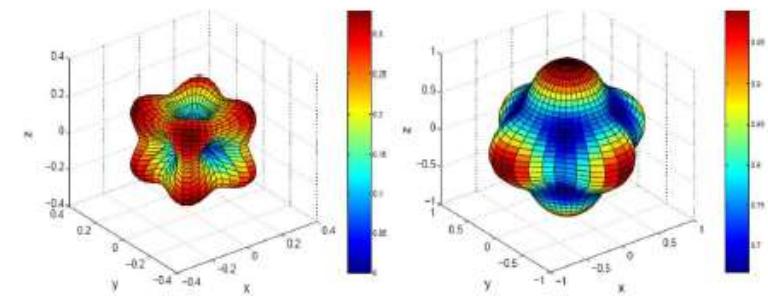
- Uniaxial anisotropy:

$$f_{an} = K_0 + K_1 \sin^2 \theta + K_2 \sin^4 \theta + K_3 \sin^6 \theta + \dots$$



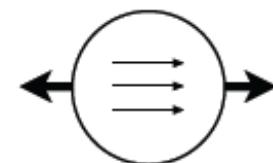
- Cubic anisotropy:

$$f_{an} = K_0 + K_1(m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2) + K_2 m_x^2 m_y^2 m_z^2 + \dots$$



magnetocrystalline

along easy axis



Anisotropy energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) \, dV$$

$$G_{an} = \int_{\Omega} f_{an}(\mathbf{m}) \, dV$$

Characteristic parameters

- The hardness parameter permits one to introduce the notion of soft versus hard material:

$$\kappa = \frac{2K_1}{\mu_0 M_s^2}$$

$\left| \begin{array}{ll} \kappa \ll 1 & \text{soft material} \\ \kappa \simeq 1 & \text{hard material} \end{array} \right.$

For iron, where $K_1 \simeq 5 \cdot 10^4 \text{ J/m}^3$ and $\mu_0 M_s \simeq 2 \text{ T}$, one finds $\kappa \simeq 0.03$.

$$l_{ex} = \sqrt{\frac{2A}{\mu_0 M_s^2}} \quad \simeq 5 \text{ nm} \quad \text{Exchange correlation length}$$

$$A \simeq 10^{-11} \text{ J/m}$$

$$l_w = \frac{l_{ex}}{\sqrt{\kappa}} = \sqrt{\frac{A}{K_1}} \quad \simeq 60 \text{ nm} \quad \text{Domain wall width}$$

$$K_1 \simeq 10^4 \text{ J/m}^3$$

$$\mu_0 M_s \simeq 1 \text{ T}$$

Magnetostatic energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) \, dV$$

Magnetic poles are sources of the magnetostatic field, that is called

Demagnetizing field inside the body

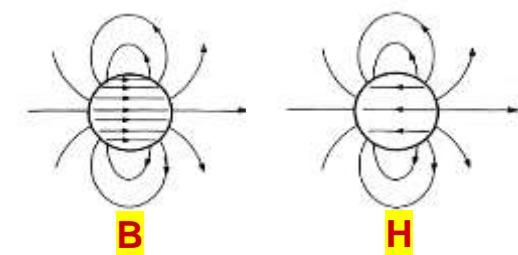
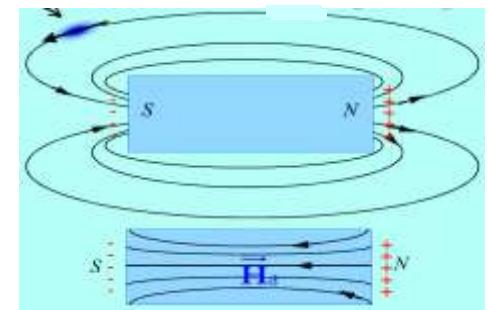
Stray field outside the body

$$\begin{cases} \nabla \cdot \mathbf{H}_m = -\nabla \cdot \mathbf{M} \\ \nabla \times \mathbf{H}_m = 0 \end{cases} \quad \mathbf{H}_m(r) = -\nabla_r \phi_m(r, r')$$

$$\mathbf{H}_m(r) = -\frac{1}{4\pi} \iiint d^3r' \nabla \cdot \mathbf{M}(r') \frac{(r - r')}{|r - r'|^3} + \frac{1}{4\pi} \iint d^2r' \mathbf{n}' \cdot \mathbf{M}(r') \frac{(r - r')}{|r - r'|^3}$$

$$\begin{cases} \rho_m = -\mu_0 \nabla \cdot \mathbf{M} \\ \sigma_m = -\mu_0 \mathbf{M} \cdot \mathbf{n} \end{cases}$$

Stray and demagnetizing field

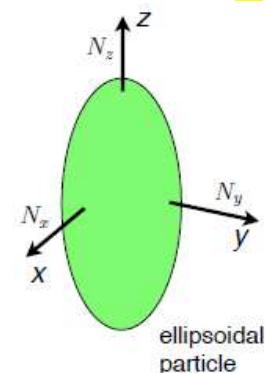


For ellipsoidal bodies, the demag field is uniform:

$$\mathbf{H}_m = \mathbf{H}_d = -NM$$

$$\overset{\leftrightarrow}{N} = \text{diag}(N_x, N_y, N_z)$$

$$N_x + N_y + N_z = 1$$



Magnetostatic energy

$$G_L(M, H_a, T) = \iiint_{\Omega} (G_{ex} + G_{an} + G_m - \mu_0 H_a \cdot M) dV$$

- The energy of the magnetostatic field in the whole space is given by:

$$G_m = \int_{\Omega_{\infty}} \frac{1}{2} \mu_0 H_m^2 dV$$

- Using the relationship between \mathbf{B} and \mathbf{H}

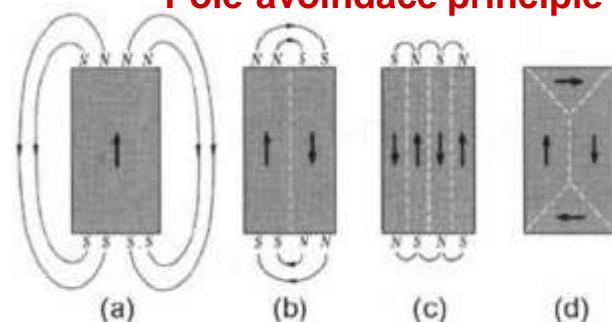
$$G_m = \int_{\Omega_{\infty}} \frac{1}{2} \mu_0 \mathbf{H}_m \cdot \left(\frac{\mathbf{B}_m}{\mu_0} - \mathbf{M} \right) dV$$

- Due to the integral orthogonality in the whole space of the solenoidal field \mathbf{B} and the conservative field \mathbf{H}_m

$$H_i = H_{\text{appl}} + H_d$$

$$G_m = - \int_{\Omega} \frac{1}{2} \mu_0 \mathbf{H}_m \cdot \mathbf{M} dV$$

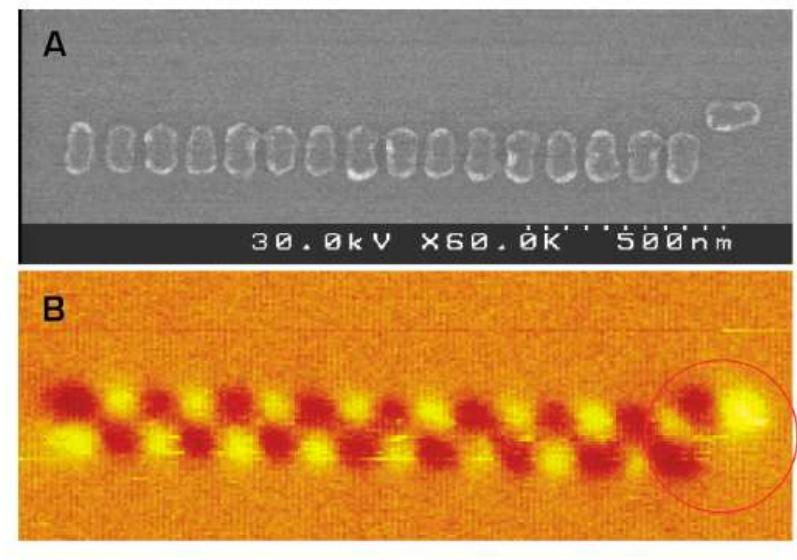
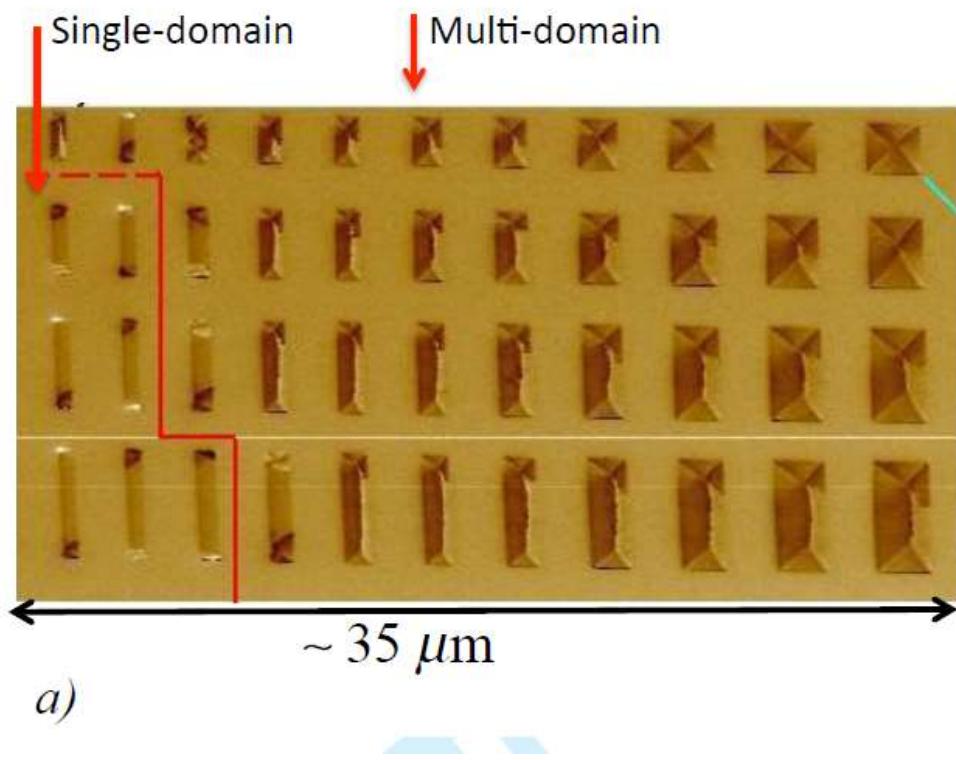
$$G_m = \frac{\mu_0 M_s^2 V}{2} (N_x m_x^2 + N_y m_y^2 + N_z m_z^2)$$



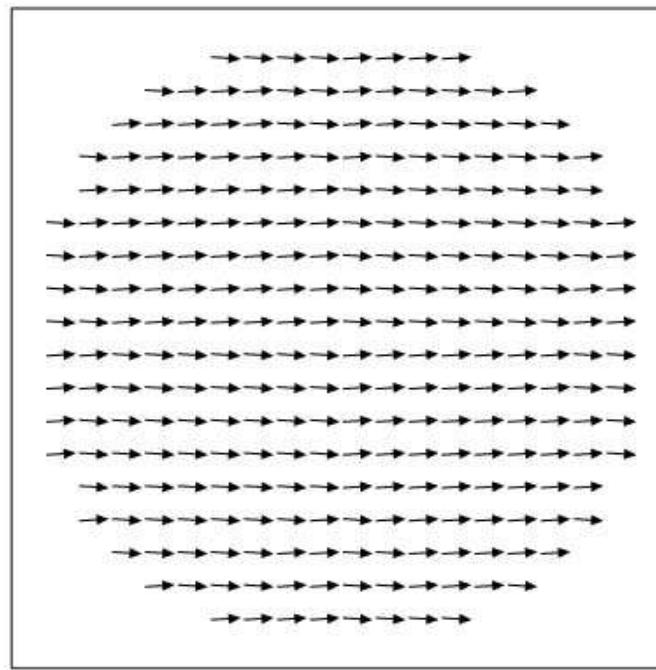
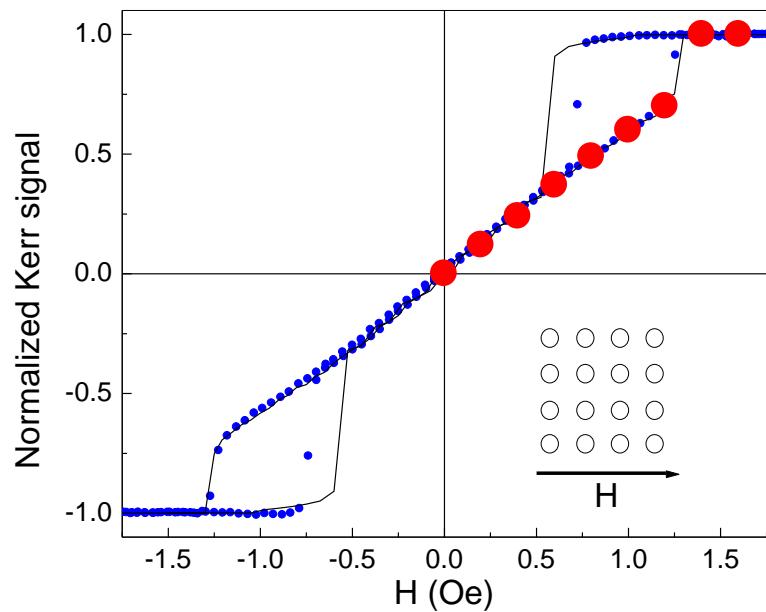
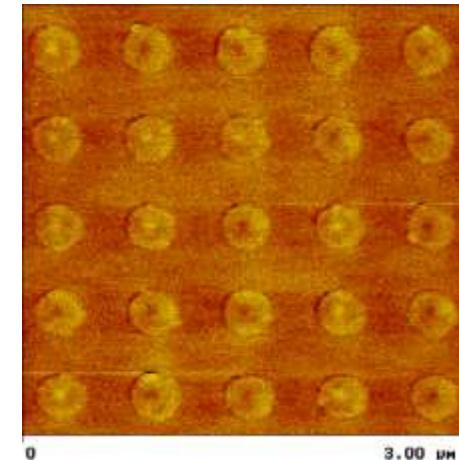
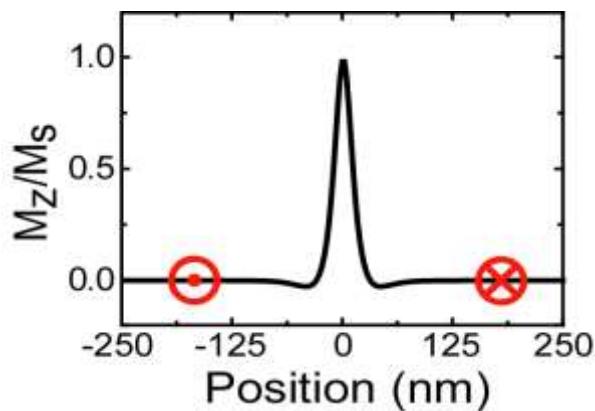
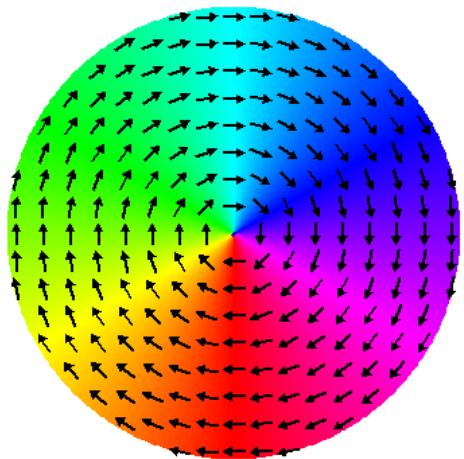
magnetostatic

zero magn. moment





Circular permalloy disks with a vortex structure



G. Gubbiotti et al., IEEE Trans. Magn. **38**, 2532 (2002).

G. Gubbiotti et al., J. Appl. Phys. **93**, 7607 (2003).

G. Gubbiotti et al., Phys. Rev. B **68**, 184409 (2003).

Looking for... Energy Minima

- The free energy functional is

$$G_L(\mathbf{M}(\cdot), \mathbf{H}_a) = \int_{\Omega} \left[A(\nabla \mathbf{m})^2 + f_{\text{an}}(\mathbf{m}) - \frac{1}{2} \mathbf{H}_m \cdot \mathbf{M} - \mu_0 \mathbf{H}_a \cdot \mathbf{M} \right] dV$$

- We introduce dimensionless quantities

$$g_L(\mathbf{m}(\cdot), \mathbf{h}_a) = \frac{G_L}{\mu_0 M_s^2 V} = \frac{1}{V} \int_{\Omega} \left[\frac{l_{\text{ex}}^2}{2} (\nabla \mathbf{m})^2 + f_{\text{an}}(\mathbf{m}) - \frac{1}{2} \mathbf{h}_m \cdot \mathbf{m} - \mathbf{h}_a \cdot \mathbf{m} \right] dV$$

$$\mathbf{h}_m = \frac{\mathbf{H}_m}{M_s} \quad , \quad \mathbf{h}_a = \frac{\mathbf{H}_a}{M_s} \quad \quad \mathbf{m}(\mathbf{r}) = \frac{\mathbf{M}(\mathbf{r})}{M_s} \quad , \quad |\mathbf{m}| = 1$$

Equilibria are stationary points of the free energy under the micromagnetic constraint

$$\delta g_L = 0 \quad \quad \quad |\mathbf{m}| = 1$$

Brown's equations

$$\delta G_L = 0 \text{ for every arbitrary variation } \delta \mathbf{v}(\mathbf{r})$$



$$\mathbf{m} \times \mathbf{H}_{\text{eff}} = 0$$

everywhere
inside the body

$$\frac{\partial \mathbf{m}}{\partial n} = 0$$

everywhere at
the body surface

Brown's
equations

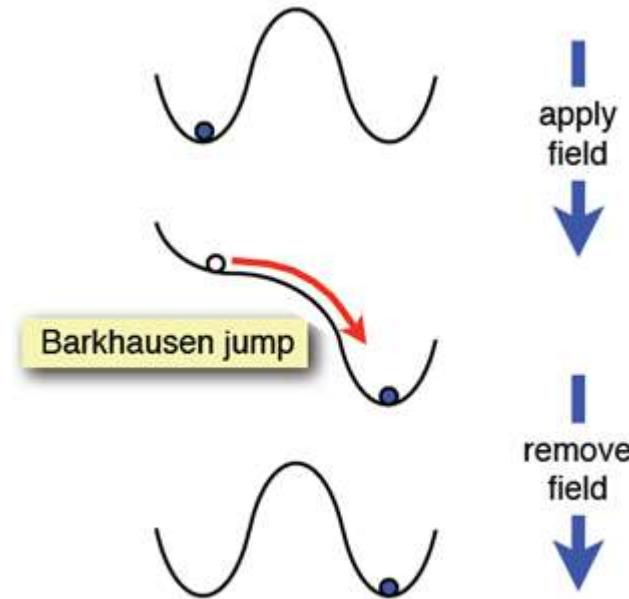
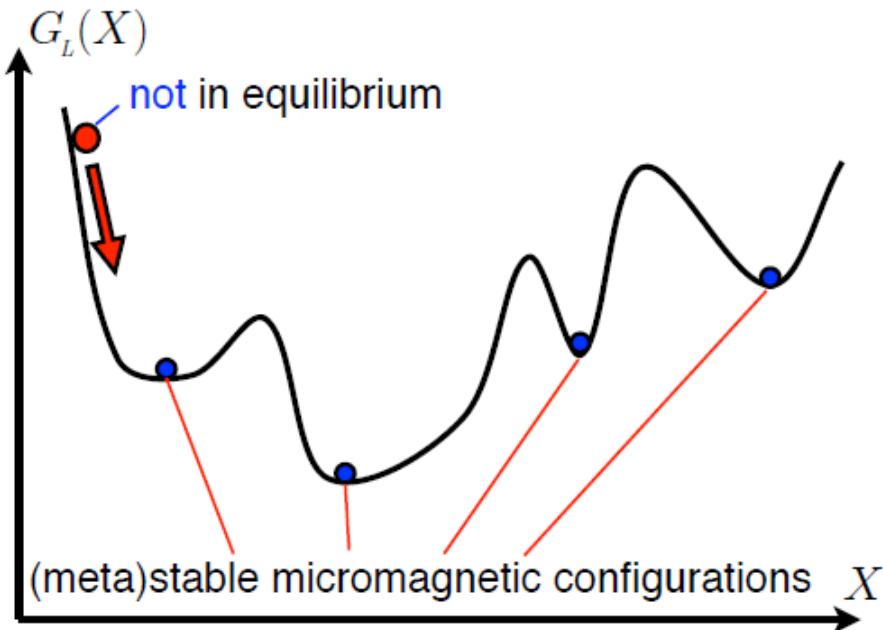
$$\mathbf{H}_{\text{eff}} = \frac{2A}{\mu_0 M_s} \nabla^2 \mathbf{m} - \frac{1}{\mu_0 M_s} \frac{\partial f_{AN}}{\partial \mathbf{m}} + \mathbf{H}_M + \mathbf{H}_a$$

exchange field anisotropy field magnetostatic field external field

effective
field

Brown's equation are strongly nonlinear functional equations, therefore in most cases it is not possible to have solution in closed form.
Nonlinear integral-partial differential equation

Energy landscape and Hysteresis



- G_L is a thermodynamic potential the **decreases as a function of time** for any transformation under constant H_a and T
- The use of energy landscapes implies a separation of time scales: the relaxation time after which the system reaches equilibrium with respect to a particular value of X is much shorter than the time over which the system evolves from one value of X to another.
- **History dependence:** the initial and final energy profiles are the same but the state occupied by the system is different depending on past history.

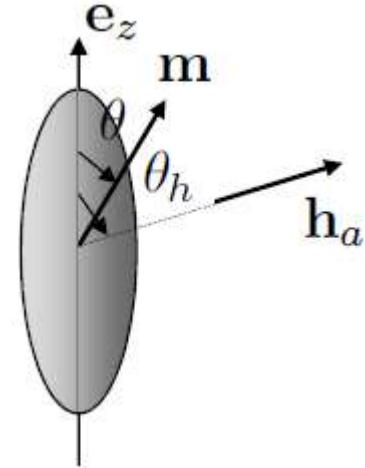
Ellipsoidal particle

Stoner-Wohlfarth model

For a generic ellipsoidal particle

$$g_{\text{ex}}(\mathbf{m}) = 0 \quad g_m(\mathbf{m}) = \frac{1}{2}N_x m_x^2 + \frac{1}{2}N_y m_y^2 + \frac{1}{2}N_z m_z^2 \quad . \quad g_a(\mathbf{m}) = -\mathbf{m} \cdot \mathbf{h}_a$$

$$g_L(\mathbf{m}) = \frac{1}{2}N_x m_x^2 + \frac{1}{2}N_y m_y^2 + \frac{1}{2}N_z m_z^2 - \mathbf{m} \cdot \mathbf{h}_a \quad .$$



Ellipsoid of rotation and intrinsic uniaxial anisotropy

anisotropy	magnetostatic+Zeeman	
$g_L(\mathbf{m}) = \frac{K_1}{\mu_0 M_s^2} (1 - m_z^2) + \frac{1}{2}N_\perp (m_x^2 + m_y^2) + \frac{1}{2}N_z m_z^2 - \mathbf{m} \cdot \mathbf{h}_a$		$m_z = \cos \theta$

$$k_{\text{eff}} = N_\perp + \frac{2K_1}{\mu_0 M_s^2} - N_z$$

$$\begin{aligned} g(\theta, h_a, \theta_h) &= -\frac{1}{2}k_{\text{eff}} \cos^2 \theta - h_a \cos(\theta_h - \theta) = \\ &= -\frac{1}{2}k_{\text{eff}} \cos^2 \theta - h_{az} \cos \theta - h_{a\perp} \sin \theta \end{aligned}$$

Multistability

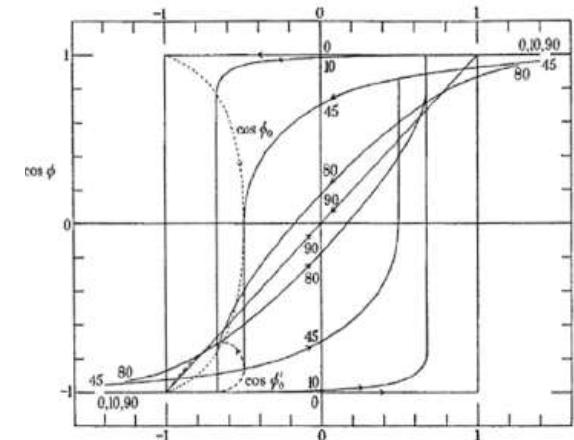
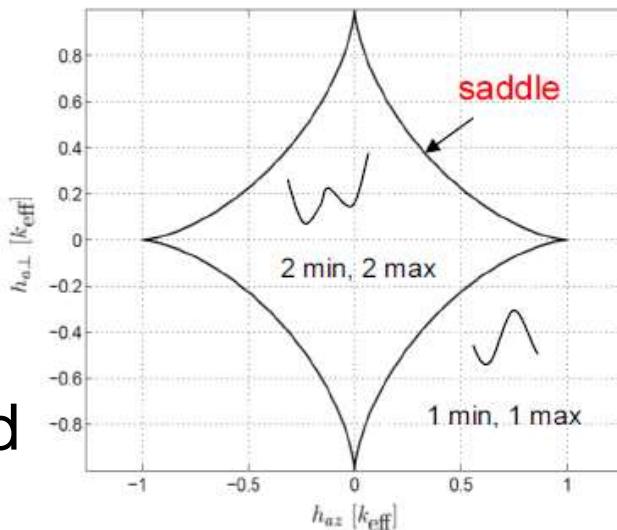
$$\frac{\partial g}{\partial \theta} = 0 \quad , \quad \frac{\partial^2 g}{\partial \theta^2} = 0$$

$$\begin{cases} \frac{h_{a\perp}}{\sin \theta} - \frac{h_{az}}{\cos \theta} = k_{\text{eff}} \\ \frac{h_{a\perp}}{\sin^3 \theta} + \frac{h_{az}}{\cos^3 \theta} = 0 \end{cases}$$

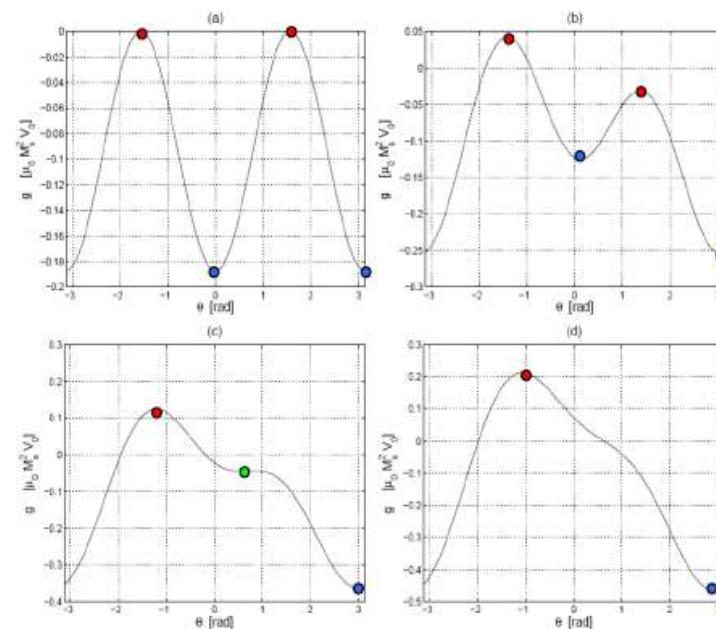


$$\begin{cases} h_{az} = -k_{\text{eff}} \cos^3 \theta \\ h_{a\perp} = k_{\text{eff}} \sin^3 \theta \end{cases}$$

SW astroid



E.C. Stoner, E.P. Wohlfarth, A mechanism of magnetic hysteresis in heterogeneous alloys, Philos. Trans. R. Soc. London Ser. A, vol. 240, pag. 599 (1948).

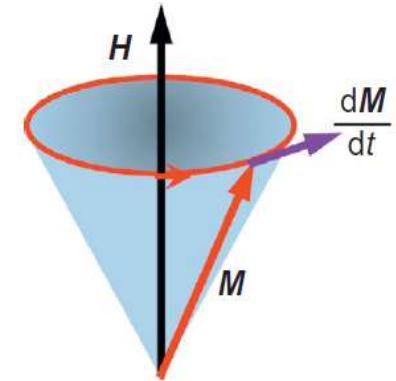


Dynamic Equation

Continuum precessional equation (Landau, 1935):

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}}$$

$$\mathbf{H}_{\text{eff}} = \frac{2}{\mu_0 M_s} \nabla \cdot (A \nabla \mathbf{m}) - \frac{1}{\mu_0 M_s} \frac{\partial f_{\text{an}}}{\partial \mathbf{m}} + \mathbf{H}_m + \mathbf{H}_a \quad \mathbf{H}_{\text{exc}} = \frac{2}{\mu_0 M_s} \nabla \cdot (A \nabla \mathbf{m}) \quad \mathbf{H}_{\text{an}} = \frac{1}{\mu_0 M_s} \frac{\partial f_{\text{an}}}{\partial \mathbf{m}}$$



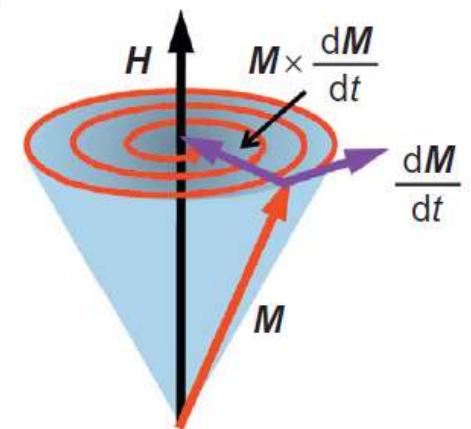
In the problem, there exist a characteristic **field scale**, given by the saturation magnetization M_s (a typical value is $\mu_0 M_s \sim 1 \text{ T}$, i.e., $M_s \sim 10^6 \text{ A/m}$) and a characteristic **time scale**, given by $(\gamma M_s)^{-1}$ ($(\gamma M_s)^{-1} \sim 6 \text{ ps}$ when $\mu_0 M_s \sim 1 \text{ T}$)

LL

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\lambda}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

LLG

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}$$



- Normalized LLG equation:

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}$$

$\mathbf{m} = \mathbf{M}/M_s$, $\mathbf{h}_{\text{eff}} = \mathbf{H}_{\text{eff}}/M_s$, time is measured in units of $(\gamma M_s)^{-1}$.

$$\alpha = 10^{-3} \div 10^{-2} \quad \text{Very small dissipation!}$$

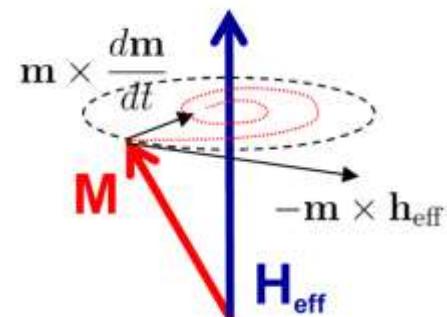
The equation is consistent with Brown's equations, because when $\partial \mathbf{m} / \partial t = 0$ $\mathbf{m} \times \mathbf{h}_{\text{eff}} = 0$

If in addition one measures lengths in units of the exchange length $l_{\text{ex}} = \sqrt{2A/\mu_0 M_s^2}$, one obtains for the effective field the dimensionless expression:

$$\mathbf{h}_{\text{eff}} = \nabla^2 \mathbf{m} + \mathbf{h}_{\text{AN}} + \mathbf{h}_M + \mathbf{h}_a$$

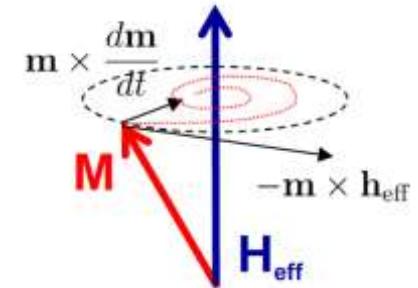
lengths are measured in units of the exchange length $l_{\text{ex}} = \sqrt{2A/\mu_0 M_s^2}$

- Solutions in closed form can be found only under restrictive assumption (later we will see an example)
- The general method of solution is numerical simulation



- Magnetization magnitude preservation:

$$\mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial t} = 0 \Rightarrow \frac{\partial |\mathbf{m}|^2}{\partial t} = 0$$

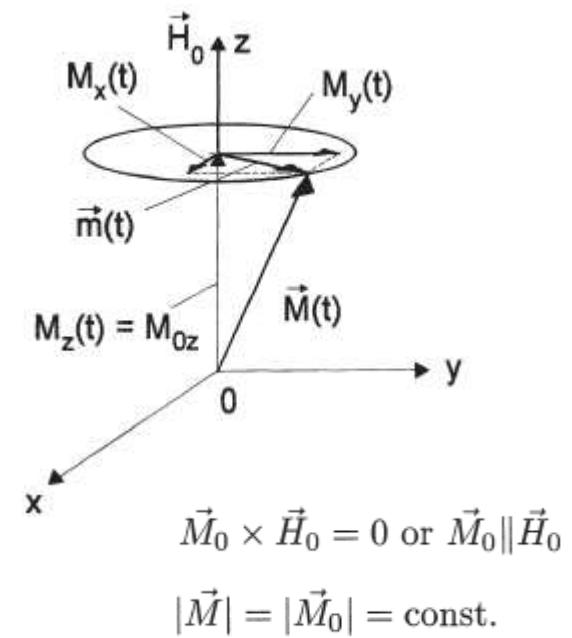


- Energy balance property:

Ferromagnetic resonance: (the free oscillator...)

Dynamic equations

$$\begin{cases} \frac{\partial M_x}{\partial t} = -\gamma \cdot \mu_0 \cdot H_0 \cdot M_y, \\ \frac{\partial M_y}{\partial t} = \gamma \cdot \mu_0 \cdot H_0 \cdot M_x, \\ \frac{\partial M_z}{\partial t} = 0 \end{cases}$$



Solution:

Free oscillator

$$\begin{cases} M_x = -R \cdot \sin(\omega_H t) \\ M_y = R \cdot \cos(\omega_H t) \end{cases}$$

$$\vec{M}(t) = M_{0z} \vec{z}_0 + \vec{m} \cdot e^{j\omega_H t}; \quad m_x^2 + m_y^2 \ll M_{0z}^2$$

$$\begin{cases} j\omega_H m_x + \gamma \mu_0 H_0 m_y = 0 \\ -\gamma \mu_0 H_0 m_x + j\omega_H m_y = 0 \\ j\omega_H m_z = 0 \end{cases} \quad m_y + jm_x = 0$$

$$\omega_H = \gamma \cdot \mu_0 \cdot H_0$$

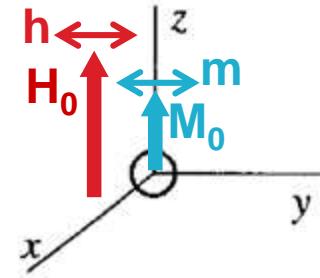
Note that here μ_0 is not included within γ

Ferromagnetic resonance: (... the forced oscillator)

$$\vec{H}(r, t) = \vec{H}_0 + \vec{h} \cdot e^{j\omega t}$$

$$|\vec{h}| \ll |\vec{H}_0|, \quad |\vec{m}| \ll |M_{0z}|$$

$$\vec{M}(r, t) = M_{0z} \vec{z}_0 + \vec{m} \cdot e^{j\omega t}$$



$$\left\{ \begin{array}{l} j\omega m_x + \omega_H m_y = \gamma \mu_0 M_0 h_y, \\ -\omega_H m_x + j\omega m_y = -\gamma \mu_0 M_0 h_x, \\ j\omega m_z = 0, \end{array} \right.$$

$$\left\{ \begin{array}{l} m_x = \frac{\gamma \mu_0 M_0 \omega_H}{\omega_H^2 - \omega^2} h_x + j \frac{\gamma \mu_0 M_0 \omega}{\omega_H^2 - \omega^2} h_y \\ m_y = -j \frac{\gamma \mu_0 M_0 \omega}{\omega_H^2 - \omega^2} h_x + \frac{\gamma \mu_0 M_0 \omega_H}{\omega_H^2 - \omega^2} h_y \\ m_z = 0 \end{array} \right.$$

$$\vec{m} = \bar{\chi} \cdot \vec{h} \quad \bar{\chi} = \begin{pmatrix} \chi & j\chi_a & 0 \\ j\chi_a & \chi & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \bar{\mu} = \mu_0 (\bar{1} + \bar{\chi})$$

$$\chi = \frac{\gamma \mu_0 M_0 \omega_H}{\omega_H^2 - \omega^2}, \quad \chi_a = \frac{\gamma \mu_0 M_0 \omega}{\omega_H^2 - \omega^2}$$

Including the «Gilbert damping»

$$\omega_H \rightarrow (\omega_H + j\omega\alpha)$$

$$\chi = \chi' + j\chi'',$$

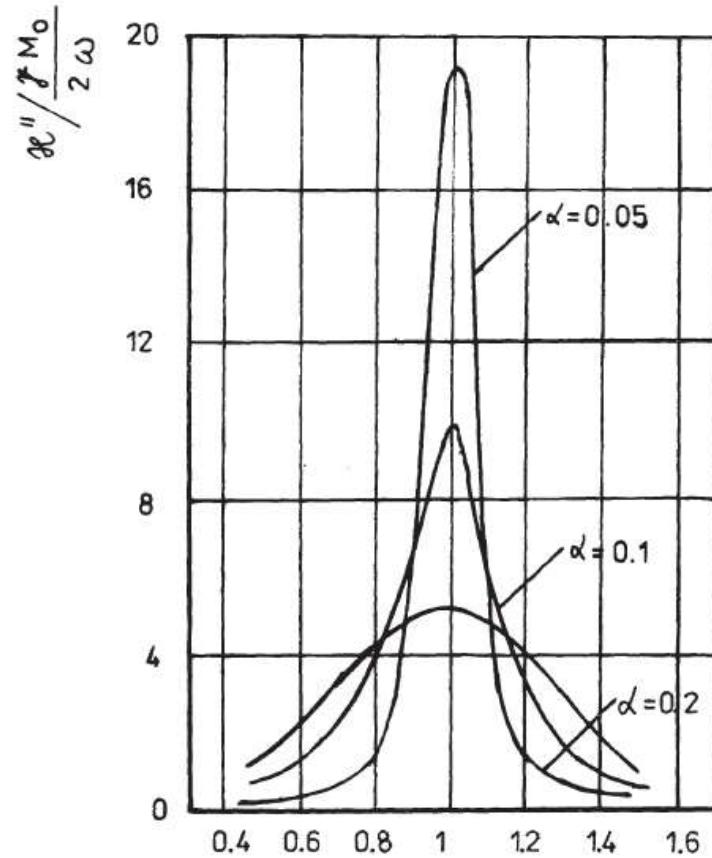
$$\chi_a = \chi'_a + j\chi''_a,$$

$$\chi' = \frac{\gamma\mu_0 M_0 \omega_H (\omega_H^2 - \omega^2)}{(\omega_H^2 - \omega^2)^2 + 4\alpha^2 \omega_H^2 \omega^2},$$

$$\chi'' = \frac{\gamma\mu_0 M_0 \omega_H \alpha (\omega_H^2 + \omega^2)}{(\omega_H^2 - \omega^2)^2 + 4\alpha^2 \omega_H^2 \omega^2},$$

$$\chi'_a = \frac{\gamma\mu_0 M_0 \omega (\omega_H^2 - \omega^2)}{(\omega_H^2 - \omega^2)^2 + 4\alpha^2 \omega_H^2 \omega^2}$$

$$\chi''_a = \frac{2\gamma\mu_0 M_0 \omega^2 \omega_H \alpha}{(\omega_H^2 - \omega^2)^2 + 4\alpha^2 \omega_H^2 \omega^2}$$



$$\|\chi''_{\max}\| = \|\chi''_{a \max}\| = \frac{\gamma\mu_0 M_0}{2\alpha\omega_H}$$

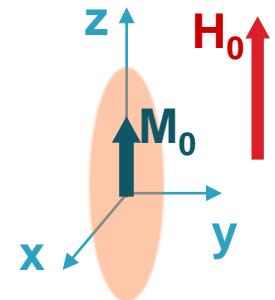
$$\omega_{\text{res}} = \omega_H \sqrt{(1 - 2\alpha^2)}$$

$$\Delta\omega_H = \alpha\omega_H, \quad \text{or} \quad \Delta H_0 = \alpha H_0$$

Ferromagnetic resonance & shape anisotropy: the internal field matters!

Let us consider an ellipsoid, magnetized along the z axis by the external field \mathbf{H}_0 , where the demagnetizing field \mathbf{H}_d is uniform:

$$\mathbf{H}_{eff} = \mathbf{H}_0 + \mathbf{H}_d$$

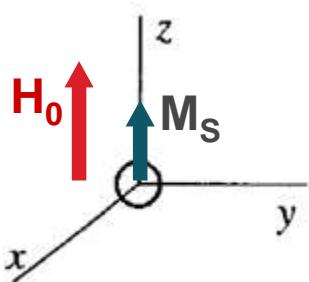


$$\omega_{res} = \gamma\mu_0 \sqrt{(H_0 + (N_y - N_z)M_s)(H_0 + (N_x - N_z)M_s)}$$

known as «Kittel formula»

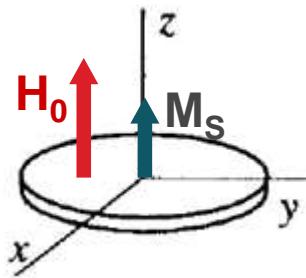
Let's have a look at a few special cases...

Ferromagnetic resonance & shape anisotropy: Relevant examples



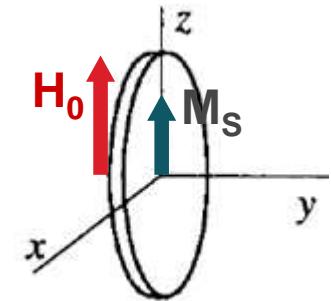
$$N_x = N_y = N_z$$

$$\omega_{FMR} = \gamma\mu_0 H_0 = \omega_0$$



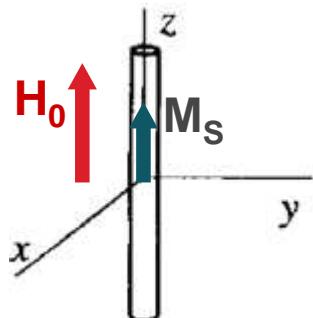
$$N_x + N_y = 0, N_z = 1$$

$$\begin{aligned}\omega_{FMR} &= \gamma\mu_0(H_0 - M_s) \\ &= \omega_0 - \omega_M\end{aligned}$$

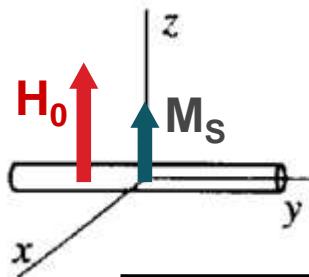


$$N_x - N_z = 0, N_y = 1$$

$$\begin{aligned}\omega_{FMR} &= \gamma\mu_0\sqrt{H_0(H_0 + M_s)} \\ &= \sqrt{\omega_0(\omega_0 + \omega_M)}\end{aligned}$$



$$\begin{aligned}\omega_{FMR} &= \gamma\mu_0\left(H_0 + \frac{M_s}{2}\right) \\ &= \omega_0 + \frac{\omega_M}{2}\end{aligned}$$



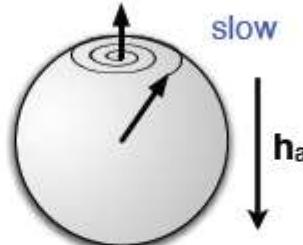
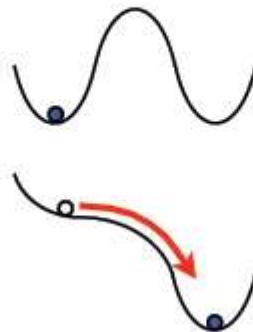
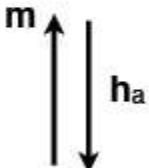
$$\begin{aligned}\omega_{FMR} &= \gamma\mu_0\sqrt{H_0\left(H_0 - \frac{M_s}{2}\right)} \\ &= \sqrt{\omega_0\left(\omega_0 - \frac{\omega_M}{2}\right)}\end{aligned}$$

$$\begin{aligned}\omega_0 &= \gamma\mu_0 H_0 \\ \omega_M &= \gamma\mu_0 M_s\end{aligned}$$

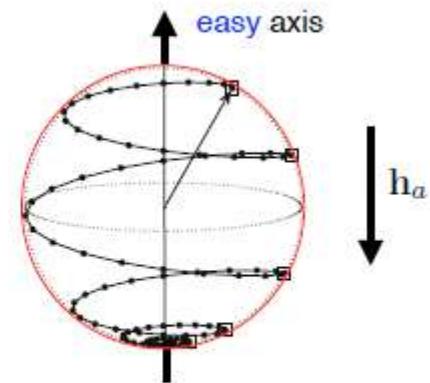
Let's go beyond such a simple MACRO-SPIN description...

Conventional Switching: climb up the barrier and slide down

- small initial torque
- long switching time
- switching is guaranteed if the field is applied for a sufficiently long time
- damping plays a dominant role

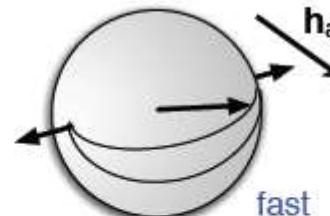
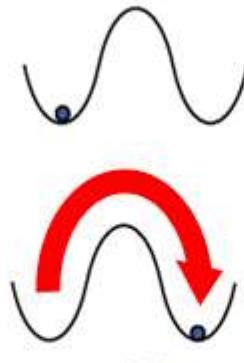
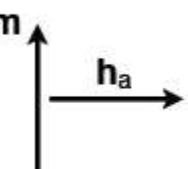


$$\frac{d\mathbf{q}_L}{dt} = -\alpha |\mathbf{m} \times \mathbf{h}_{\text{eff}}|^2$$

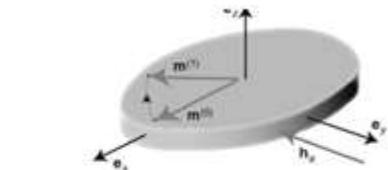


Precessional Switching: go around the barrier and STOP behind

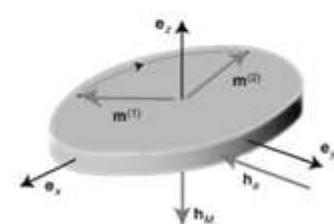
- large initial torque
- short switching time
- switching occurs only if a field pulse of suitable duration is applied
- damping plays no role (quasi-conservative dynamics)



$$\frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\text{eff}}$$



Initial torque tilts magnetization out-of-plane

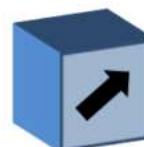
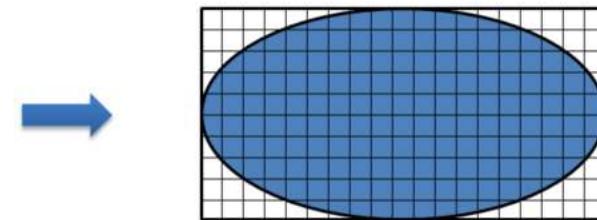
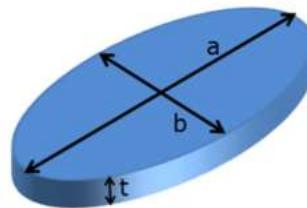
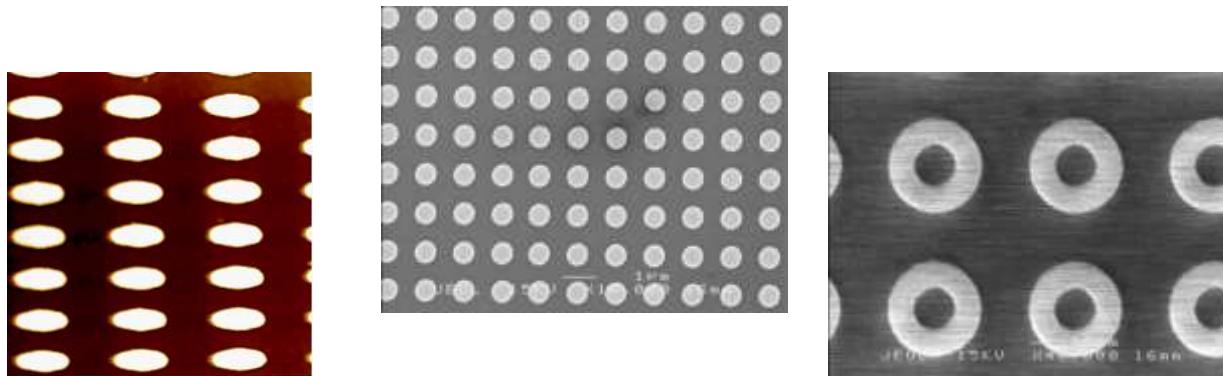


Demagnetizing field drives the precession to the reversed state

**Up to now
we have used a MACRO-SPIN description**

**Does it work properly for «real»
micro- or nano- magnetic samples?**

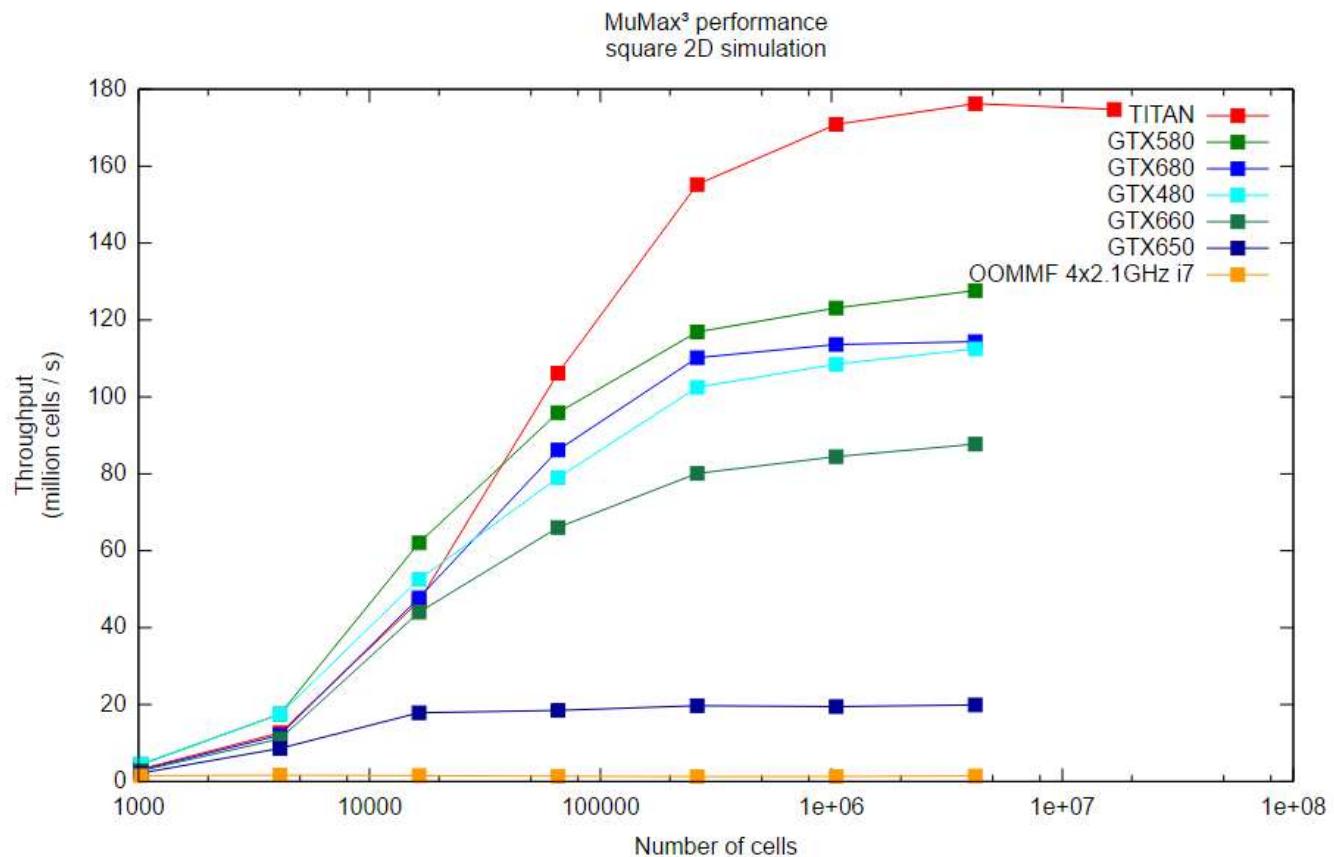
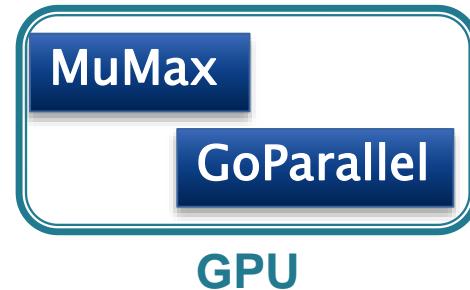
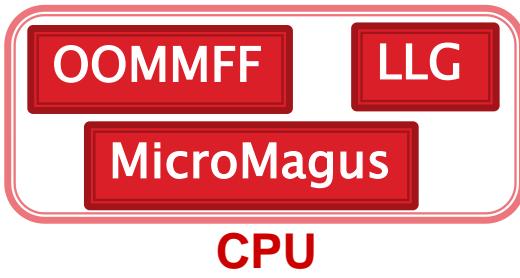
Micromagnetic approach: a “realistic” and “predictive” description of mesoscopic samples



Each cell contains a single spin:

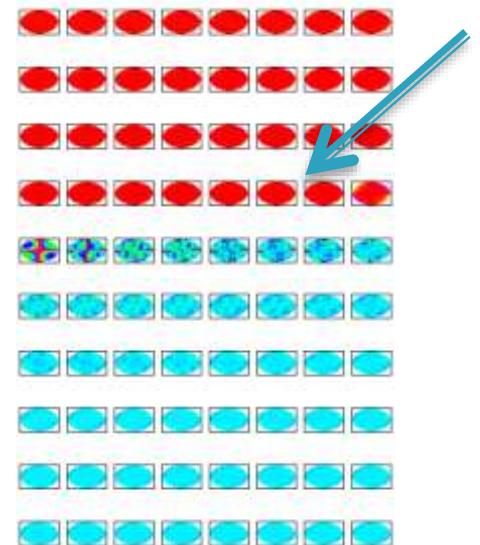
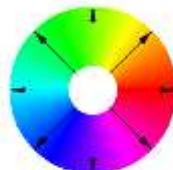
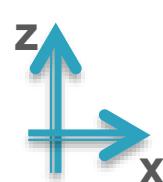
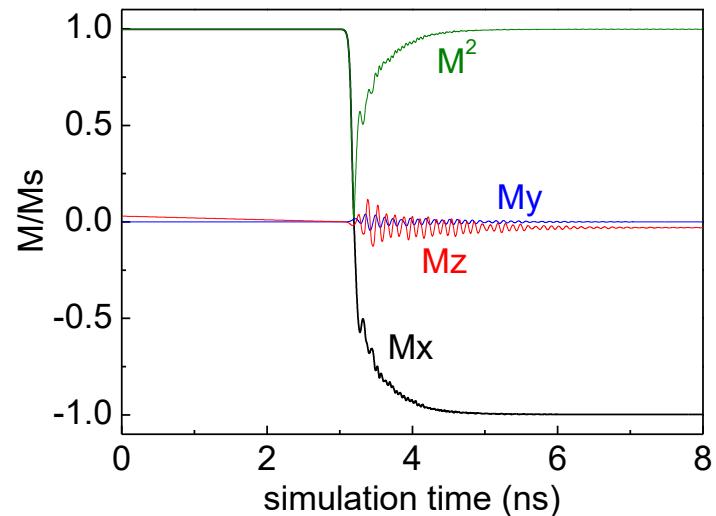
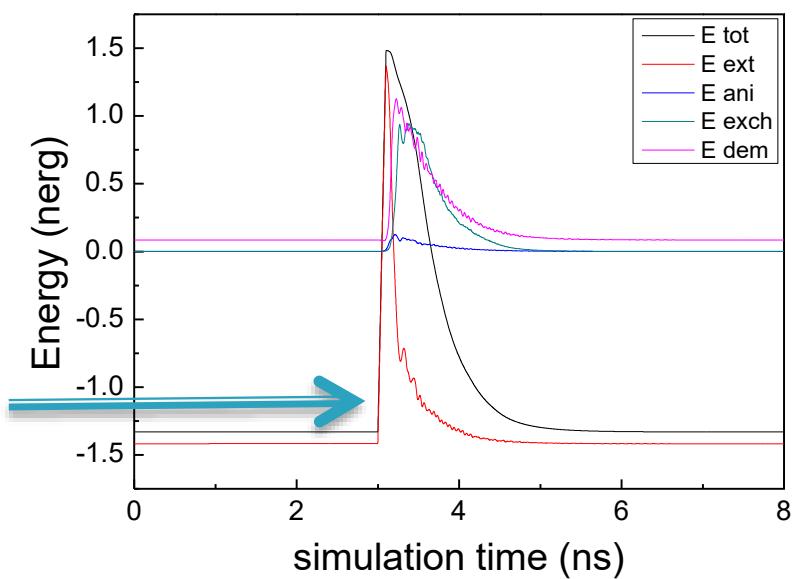
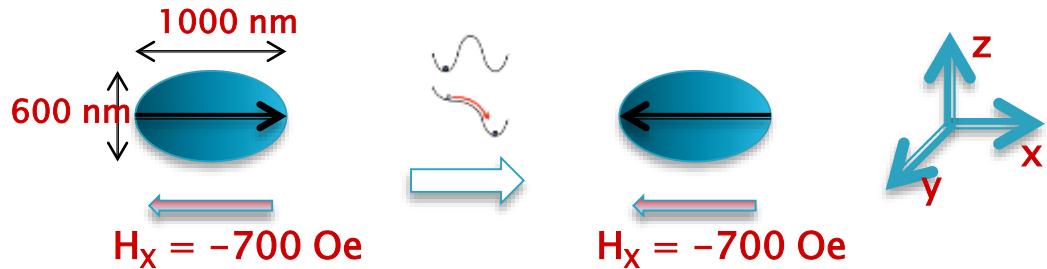
- 1) constant **modulus (M_s)** and **position**
- 2) its **orientation** in 3 dimensions may vary

Typical dimensions of the elementary cells: 1-5 nm



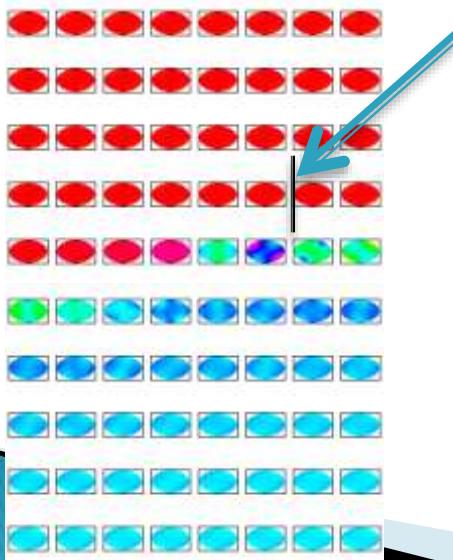
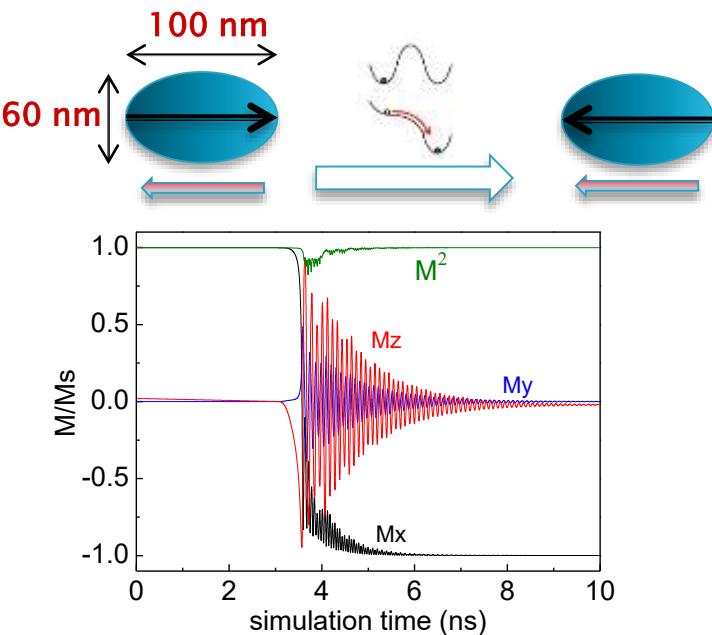
A Case Study: the planar elliptical dot

Conventional switching: elliptical microdot



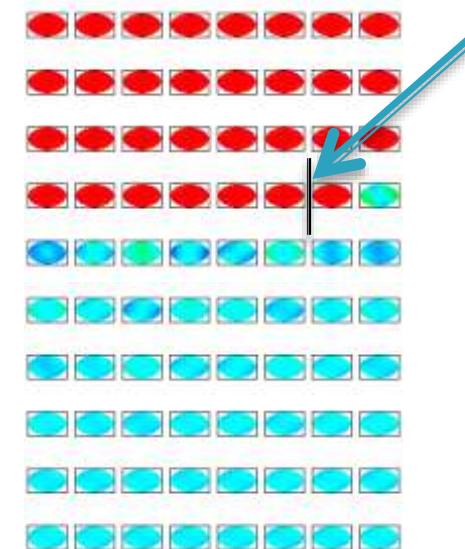
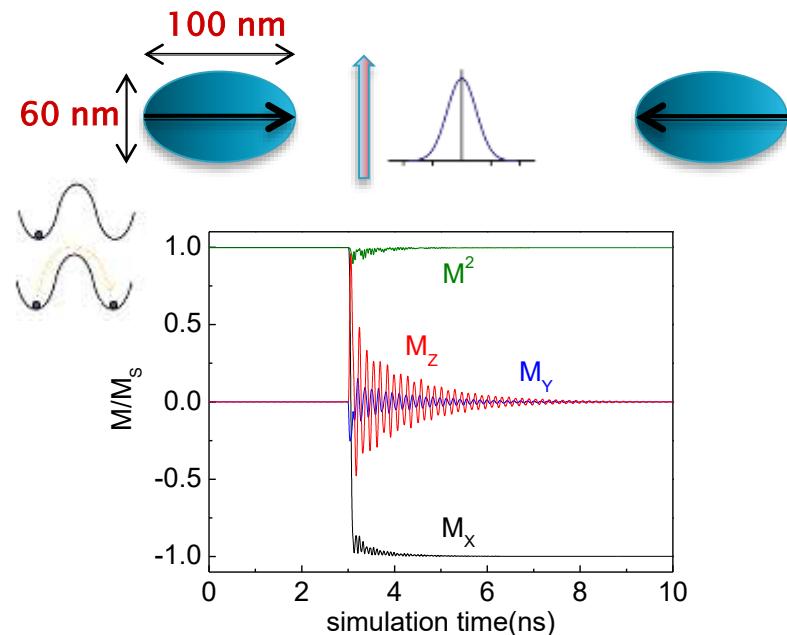
Elliptical nanodot

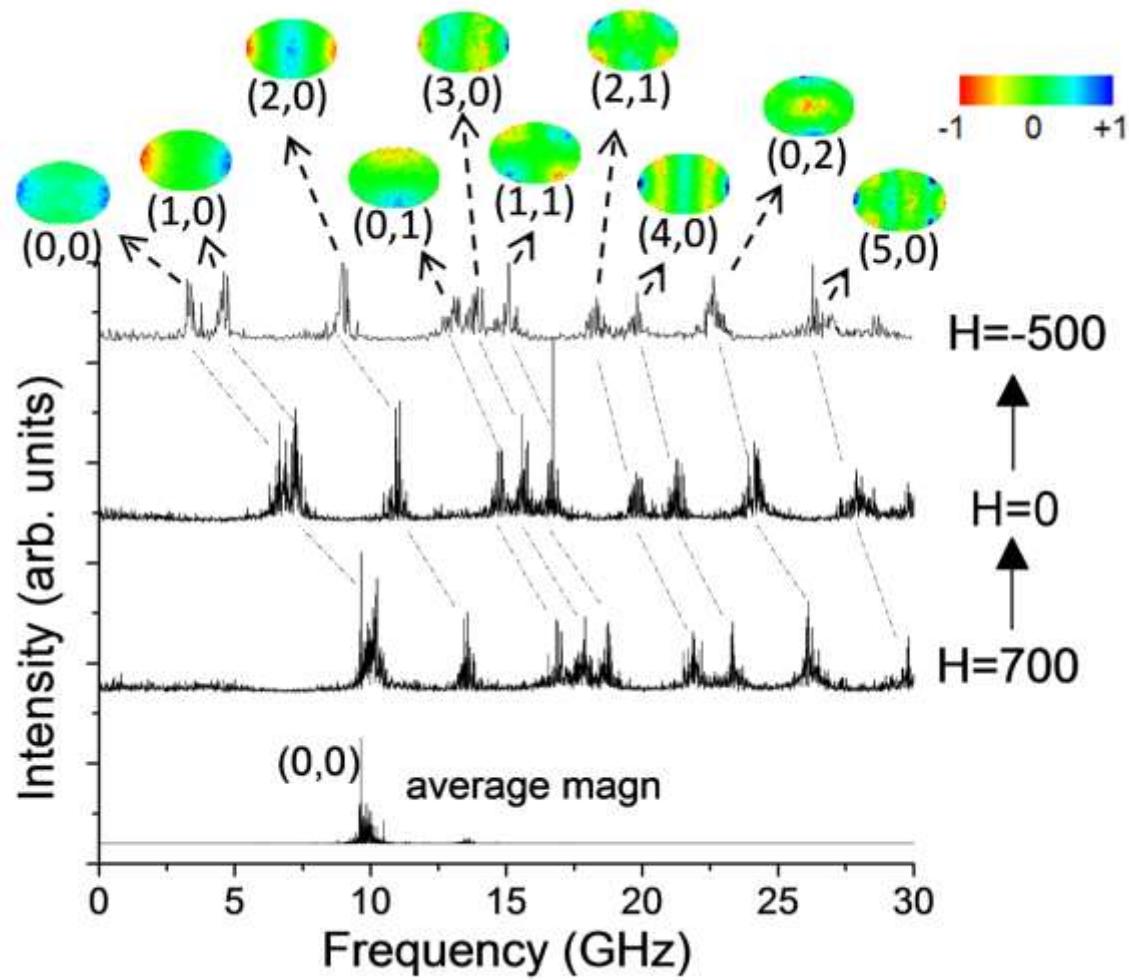
Conventional switching



vs.

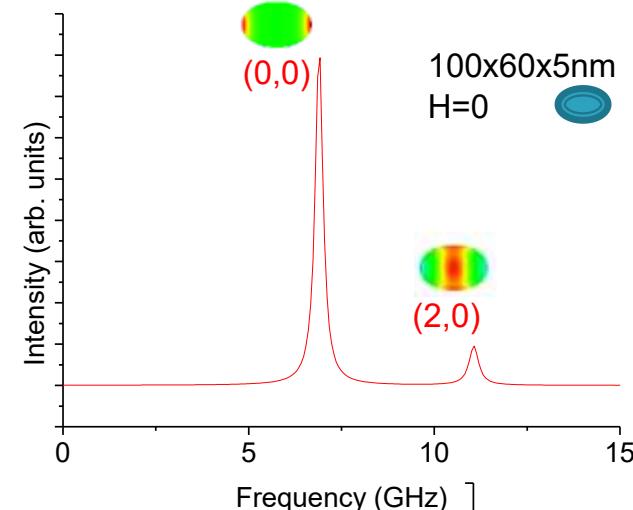
precessional switching



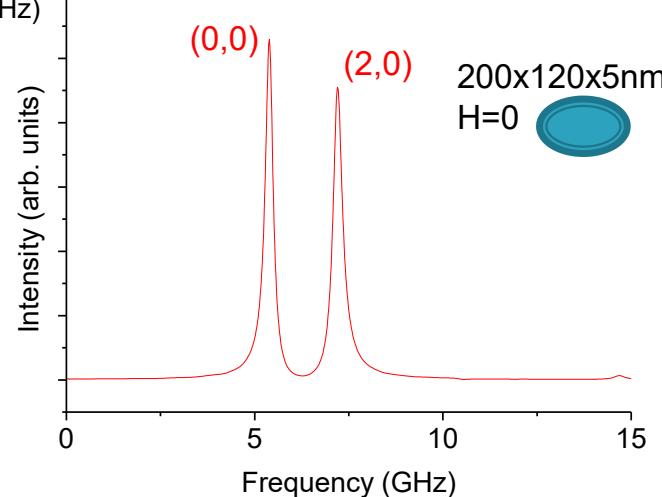


Complicated dynamics, even for relatively small objects!

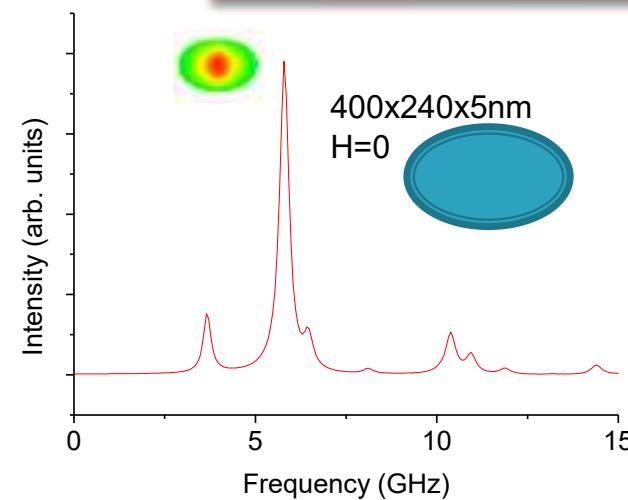
Inhomogeneity in a sub-200 nm elliptical dot



In sub-200 nm dots, the «fundamental» mode is the one at the lowest frequency (exchange-dominated modes)

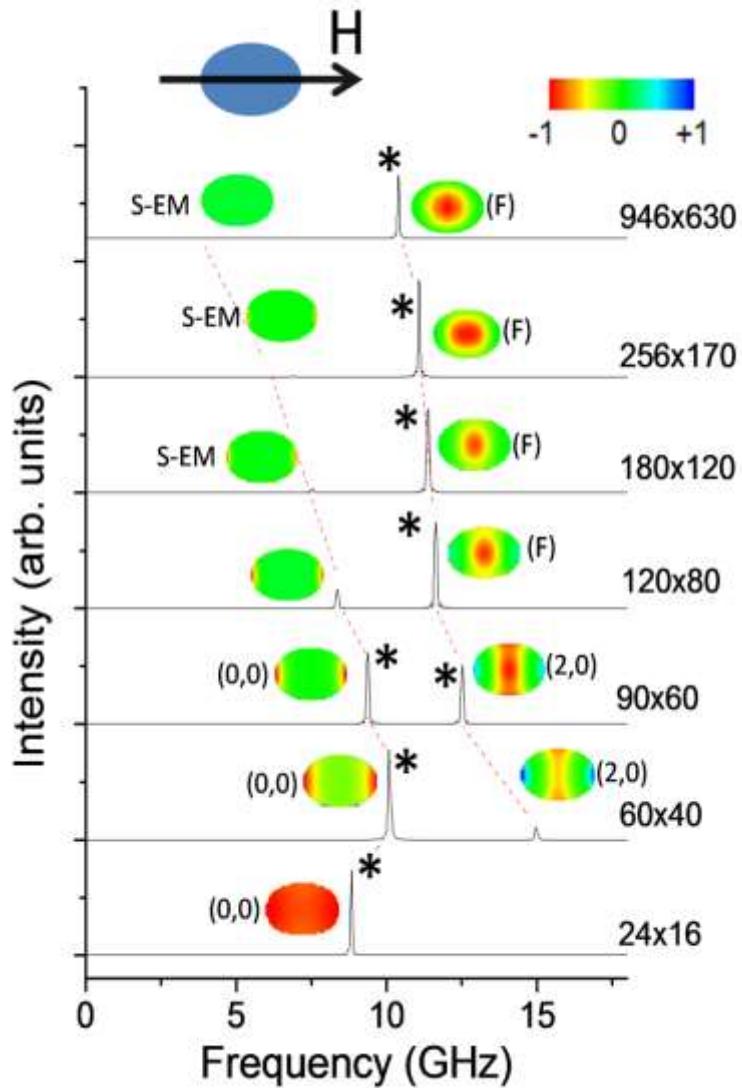


For intermediate size there are two «fundamental» modes!

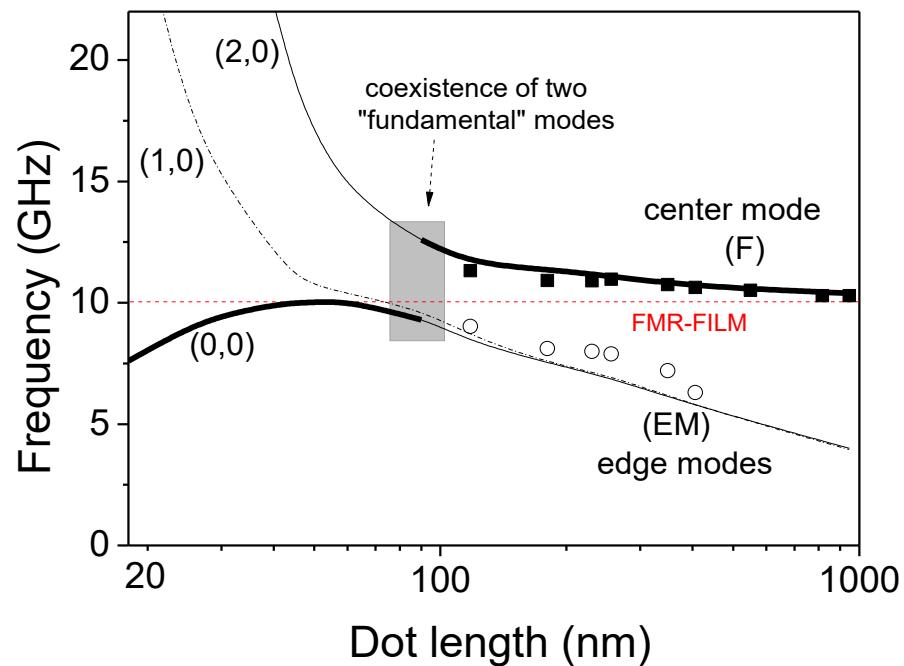


In over-200 nm dots, the «fundamental» mode is NOT the one at the lowest frequency (dipolar-dominated modes)

Effect of increasing the lateral size of the dot

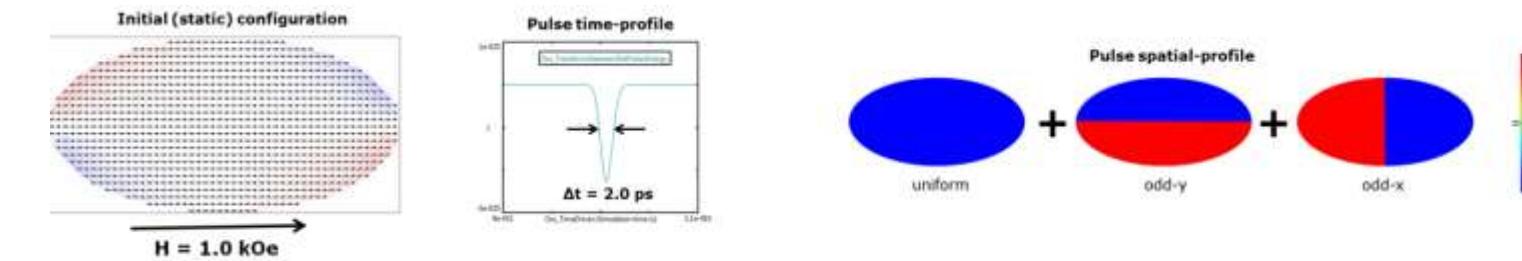


10 nm thick permalloy elliptical dots

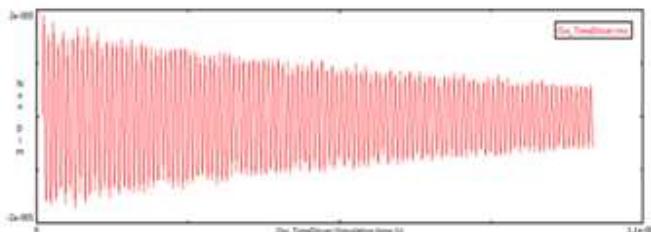


G. Carlotti, et al. “From micro- to nano- magnetic dots: evolution of the eigenmodes spectrum on reducing the lateral size”, J. Phys. D: Appl. Phys., 47, 265001 (2014)

Excitation with a “properly shaped” field pulse:



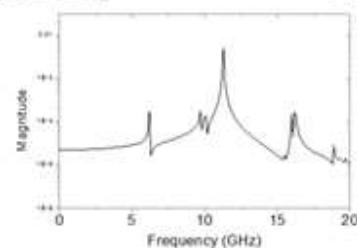
$$\bar{M}(t) = \sum_i m(r_i, t)$$



FFT

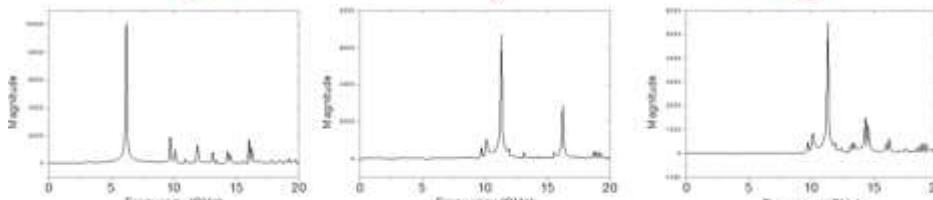
$$\bar{M}(\omega) \xrightarrow{\text{FFT}}$$

Power Spectrum (PS) of the averaged magnetization



$$m(r_i, t) \xrightarrow{\text{FFT}} m(r_i, \omega)$$

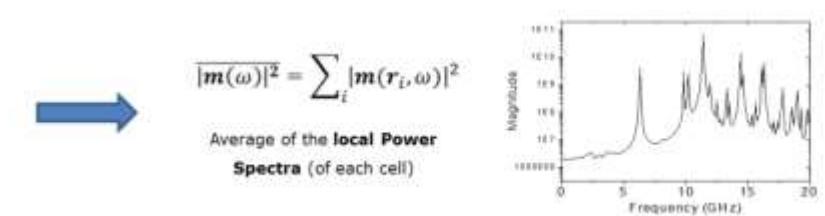
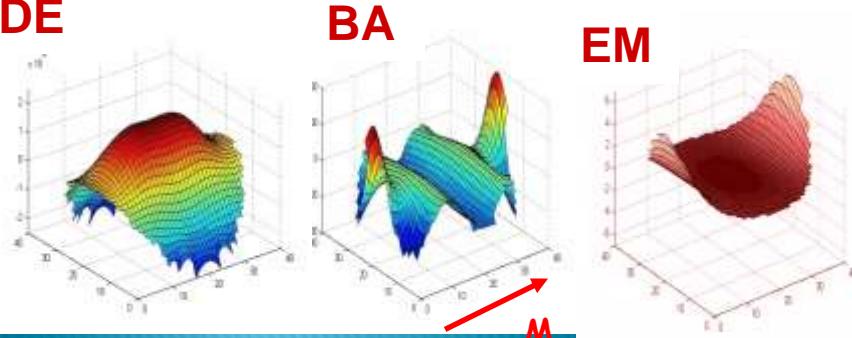
$H = 1.0 \text{ kOe}$



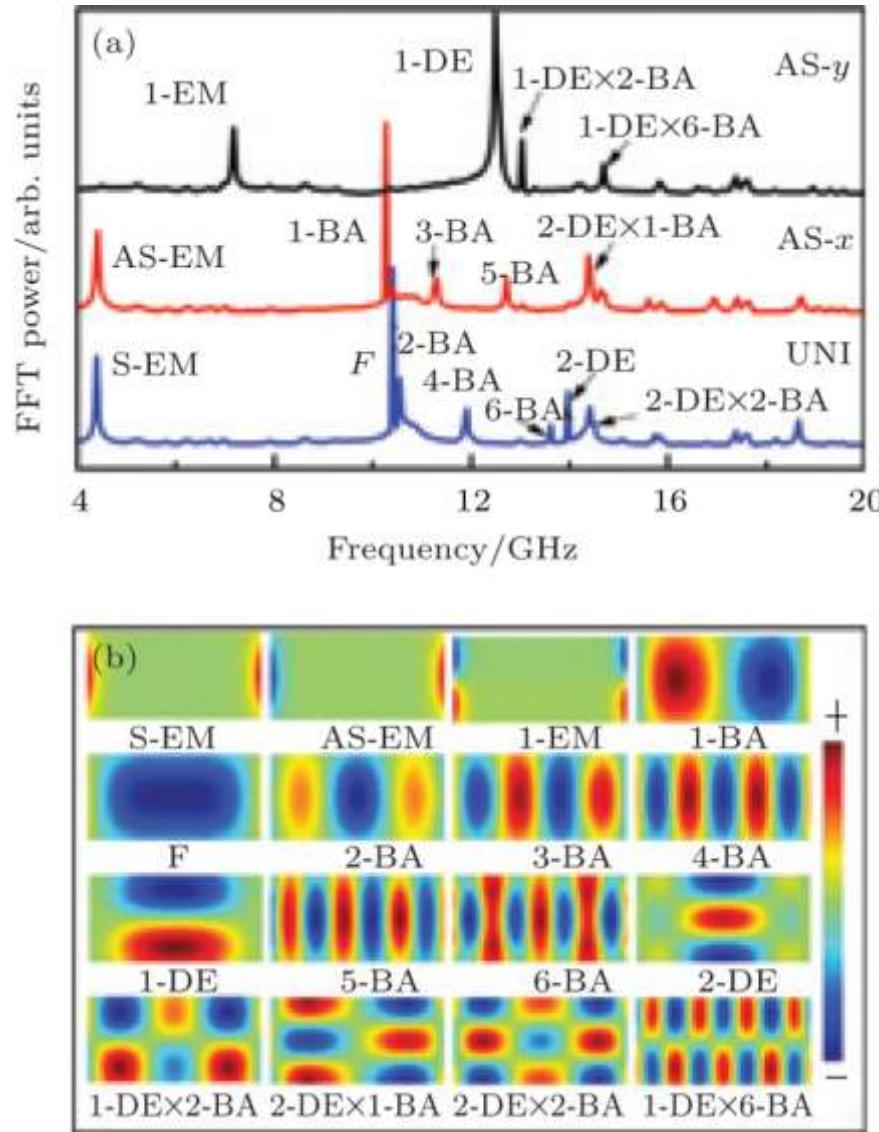
$$\xrightarrow{\text{Average of the local Power Spectra (of each cell)}}$$

$$|\bar{m}(\omega)|^2 = \sum_i |m(r_i, \omega)|^2$$

DE **BA** **EM**

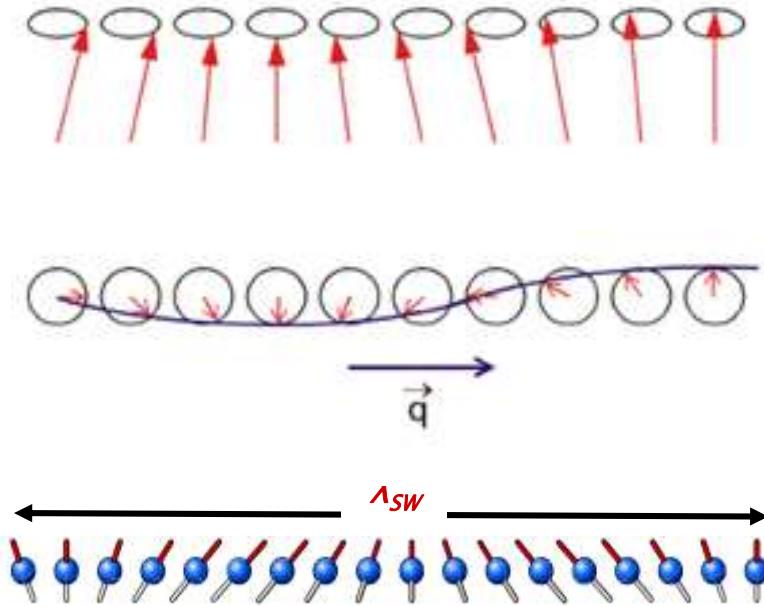


A micrometric rectangular dot



Spin waves

One dimensional chain of spin



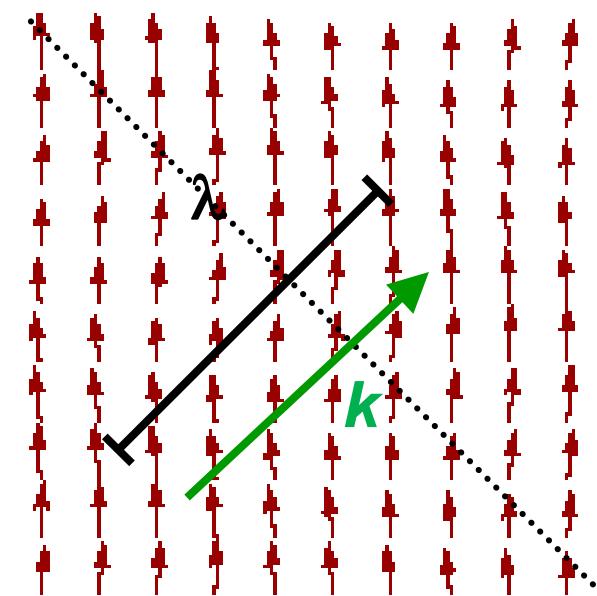
Zur Theorie des Ferromagnetismus.

Von F. Bloch, zurzeit in Utrecht.

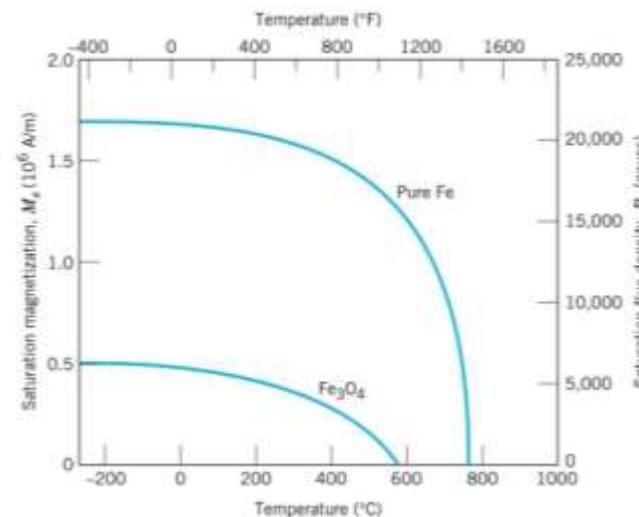
(Eingegangen am 1. Februar 1930.)

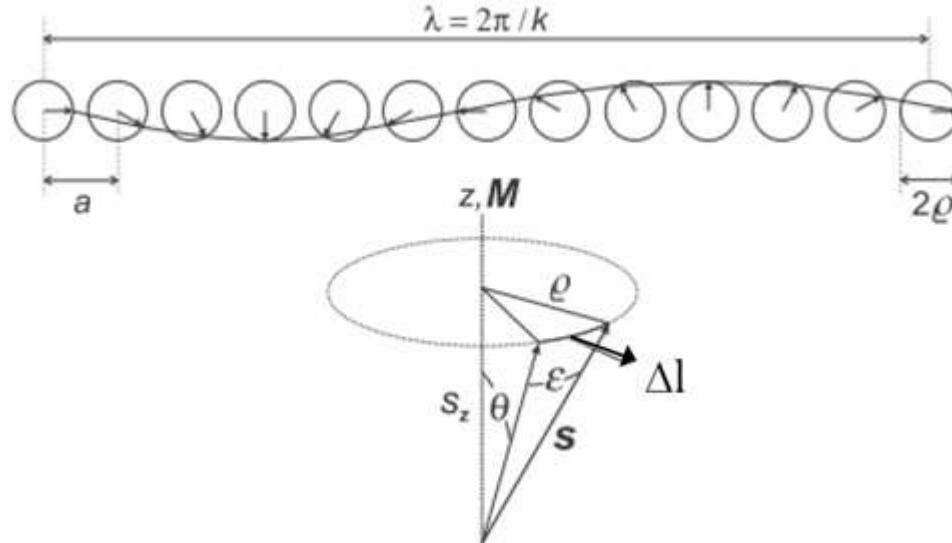
Beim Austauschvorgang der Elektronen im Kristall werden die Eigenfunktionen nullter und Eigenwerte erster Näherung für die Termsysteme hoher Multiplizität bestimmt, wobei die Kopplung zwischen Spin und Bahn vernachlässigt wird. Sie gestatten, das ferromagnetische Verhalten bei tiefen Temperaturen zu untersuchen und insbesondere die Frage zu beantworten, unter welchen Bedingungen Ferromagnetismus überhaupt möglich ist. Es zeigt sich, daß dies nur für räumliche Gitter der Fall ist; die Sättigungsmagnetisierung hat dann für tiefe Temperaturen die Form $M(T) = M(0)[1 - (T/\Theta)^{3/2}]$.

Wave front



$$M(T) = M(0) \left[\left(1 - \frac{T}{\theta_c} \right)^{3/2} \right]$$





$$\Delta l = 2\pi\rho/N = 2\pi\rho a/\lambda = \rho a k$$

$$\Delta l = \rho \sin(\varepsilon) \approx \rho \varepsilon$$

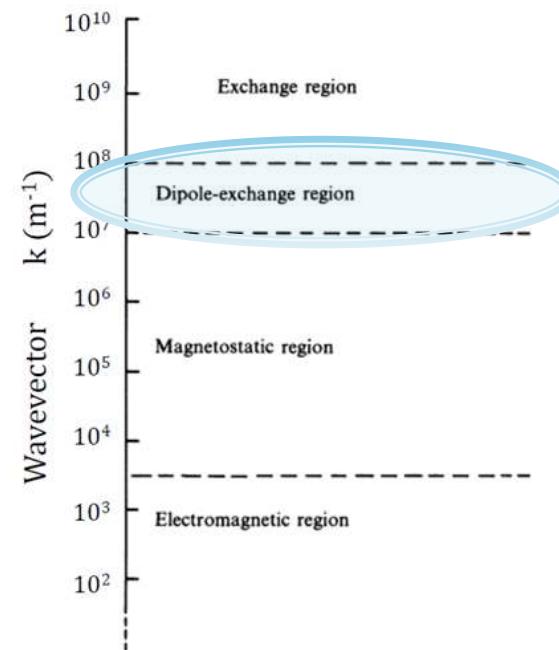
$\rightarrow \varepsilon = ak$

Heisenberg Hamiltonian for a chain of spins (1st neighbours approximation)

$$\Delta E = 2JS^2(1 - \cos \varepsilon) = 2JS^2(1 - 1 + \varepsilon^2/2) = JS^2\varepsilon^2$$

$$\Delta E = NJS^2\varepsilon^2 = NJS^2ak^2 = Dk^2$$

... but this is true only in the exchange-dominated regime!



$$\lambda \sim l_{ex} = \frac{\sqrt{2A\mu_0}}{M_s} = 3 - 10 \text{ nm}$$

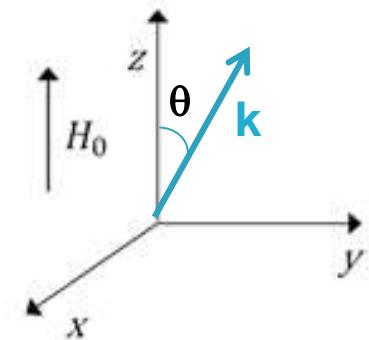
$$k = \frac{2\pi}{\lambda} = (0.6 - 3)10^9 \text{ m}^{-1}$$

Spin Waves in an infinite medium

$$\chi \sin^2 \theta = -1$$

$$k_x^2 + k_y^2 = k^2 \sin^2 \theta$$

$$k_z^2 = k^2 \cos^2 \theta$$

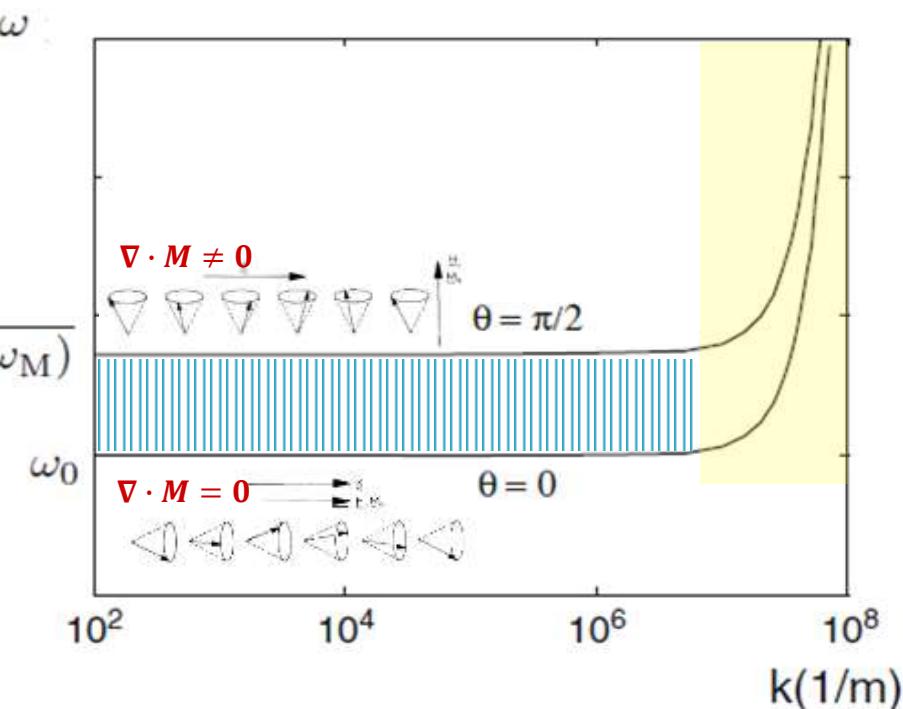
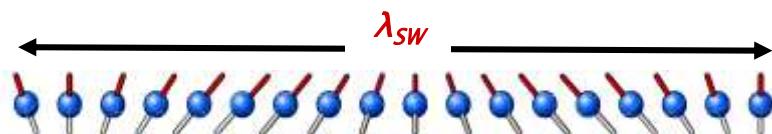


$$\omega = [\omega_0(\omega_0 + \omega_M \sin^2 \theta)]^{1/2}$$

$$\omega_0 = \gamma \mu_0 H_0$$

$$\omega_M = \gamma \mu_0 M_s$$

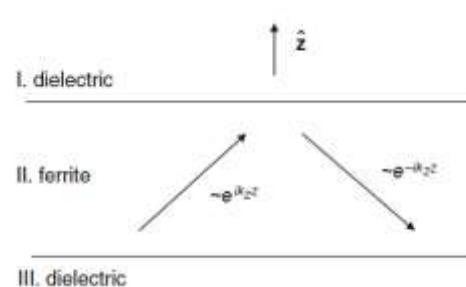
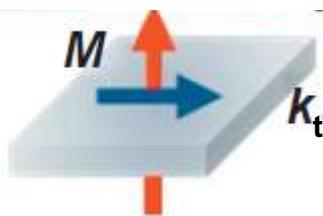
$$\sqrt{\omega_0(\omega_0 + \omega_M)}$$



Correction due
to exchange

$$\omega = [(\omega_0 + \omega_M \lambda_{\text{ex}} k^2)(\omega_0 + \omega_M (\lambda_{\text{ex}} k^2 + \sin^2 \theta))]^{1/2}$$

Magnetostatic Forward Volume Modes (MSFVW)



$$\tan \left[\frac{k_t d}{2} \sqrt{-(1+\chi)} \right] = \frac{1}{\sqrt{-(1+\chi)}}$$

$$-\cot \left[\frac{k_t d}{2} \sqrt{-(1+\chi)} \right] = \frac{1}{\sqrt{-(1+\chi)}}$$

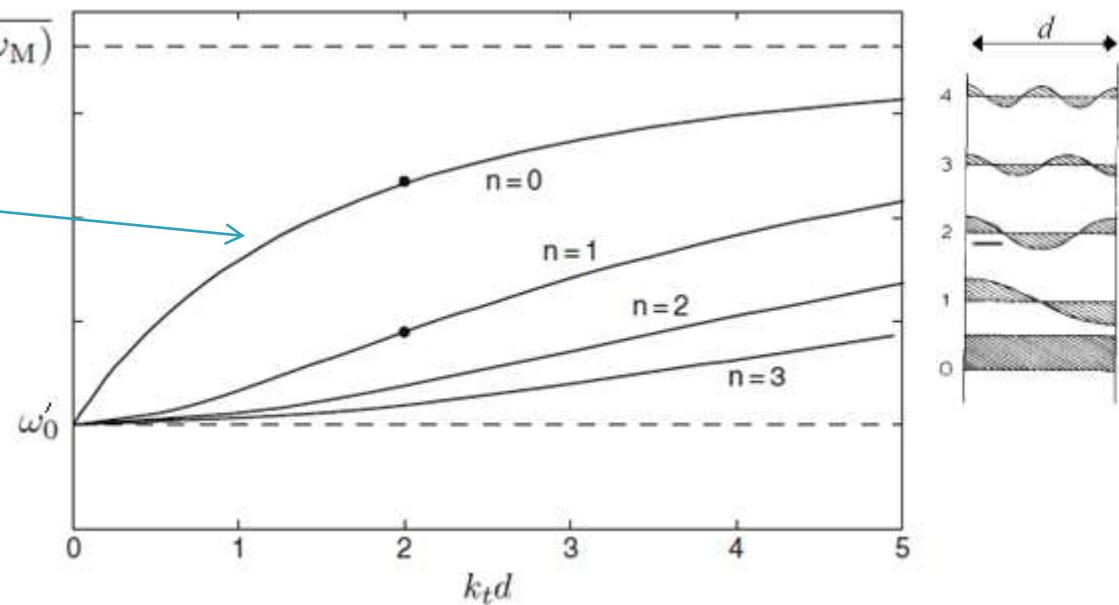
$$\omega^2 = \omega'_0 \left[\omega'_0 + \omega_M \left(1 - \frac{1 - e^{-k_t d}}{k_t d} \right) \right]$$

$$\frac{1}{v_g} \Big|_{kd=0} = \frac{4}{\omega_M d}$$

$$\begin{aligned} \omega'_0 &= \omega_{FMR} = \\ &= \gamma \mu_0 (H_a - M_s) = \omega_0 - \omega_M \end{aligned}$$

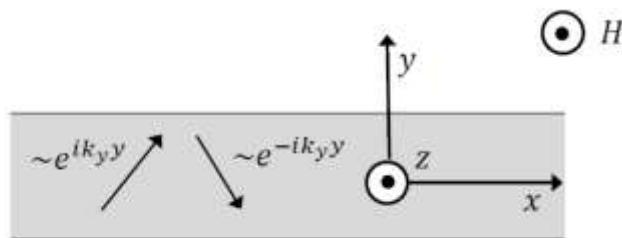
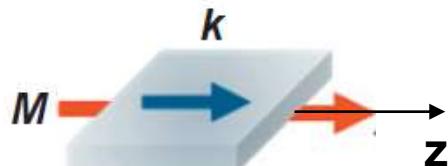
$$\omega_0 = \gamma \mu_0 H_0$$

$$\omega_M = \gamma \mu_0 M_s$$



- Same cutoff frequency
- Isotropic propagation
- Positive group velocity
- Sinusoidal amplitude distribution across the thickness

Magnetostatic Backward Volume Waves (MSBVW or BA)



$$\tan \left[\frac{k_z d}{2\sqrt{-(1+\chi)}} \right] = \sqrt{-(1+\chi)}$$

$$-\cot \left[\frac{k_z d}{2\sqrt{-(1+\chi)}} \right] = \sqrt{-(1+\chi)}$$

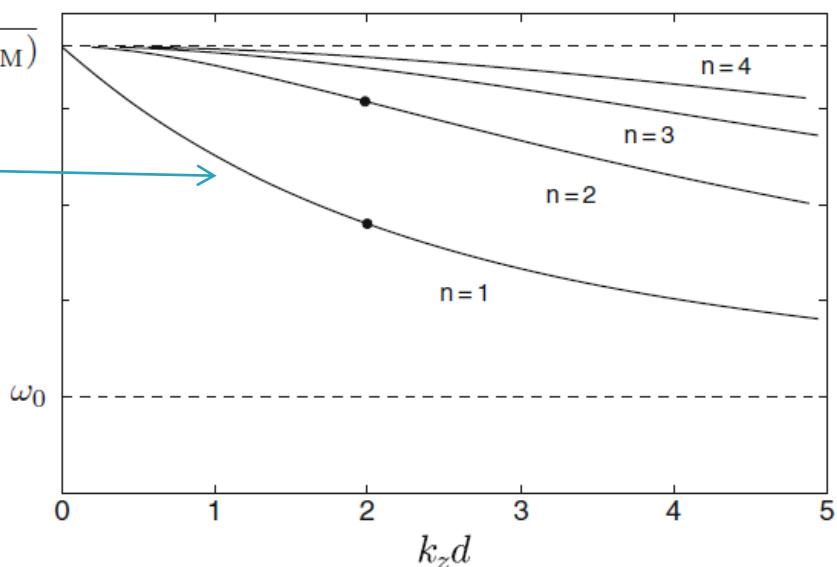
$$\omega^2 = \omega_0 \left[\omega_0 + \omega_M \left(\frac{1 - e^{-k_z d}}{k_z d} \right) \right]$$

$$\left. \frac{1}{v_g} \right|_{kd=0} = - \frac{4}{\omega_M d} \frac{\sqrt{\omega_0(\omega_0 + \omega_M)}}{\omega_0}$$

$$\begin{aligned} \omega_{FMR} &= \gamma \mu_0 \sqrt{H_a(H_a + M_s)} \\ &= \sqrt{\omega_0(\omega_0 + \omega_M)} \end{aligned}$$

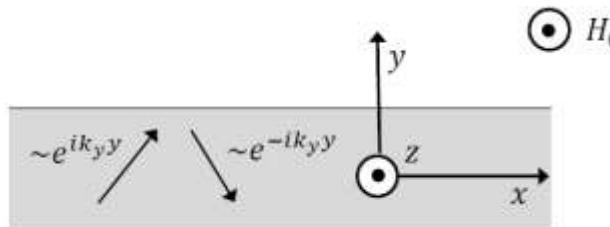
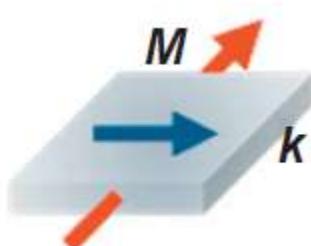
$$\omega_0 = \gamma \mu_0 H_0$$

$$\omega_M = \gamma \mu_0 M_s$$

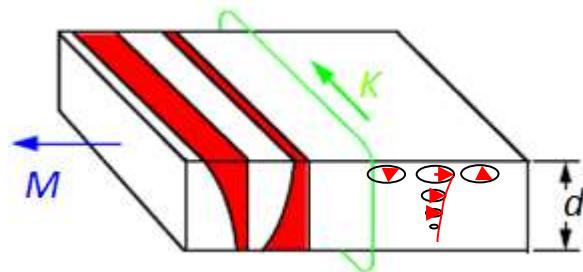


- Same cutoff frequency
- Anisotropic propagation: k/M
- Negative group velocity
- Sinusoidal amplitude distribution across the thickness

Magnetostatic Surface Waves (MSSW) or Damon-Eshbach (DE)

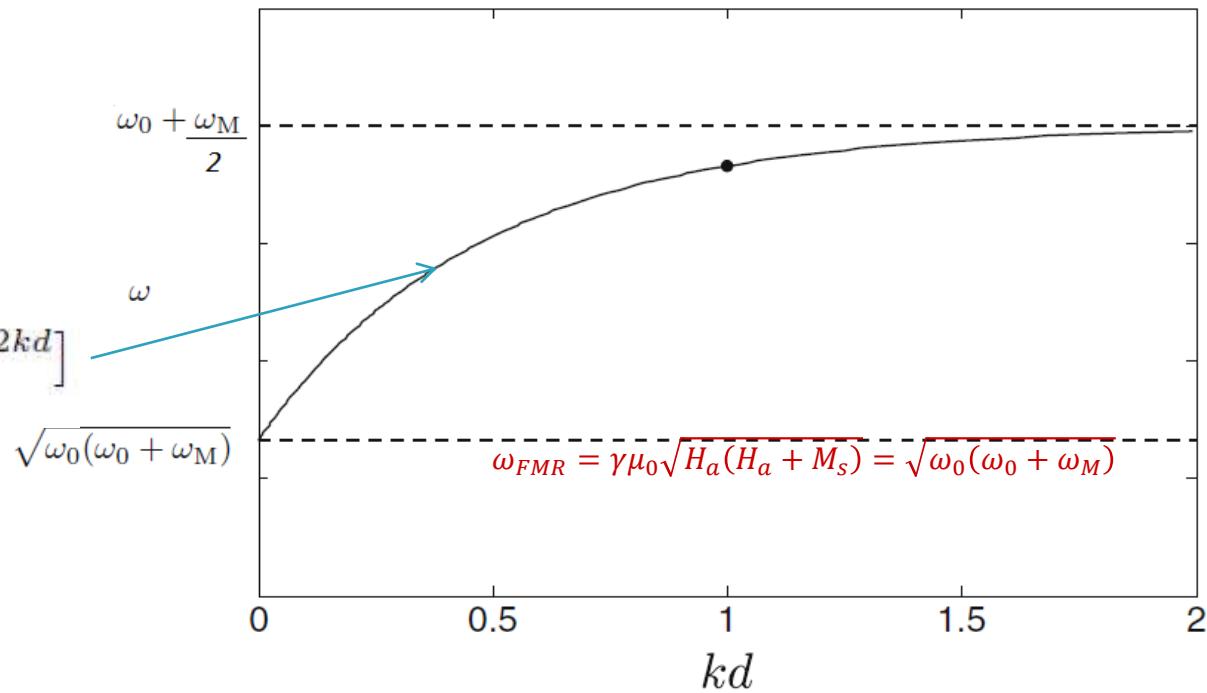


$$|k_y| = k_x \equiv k$$



$$\omega^2 = \omega_0(\omega_0 + \omega_M) + \frac{\omega_M^2}{4} [1 - e^{-2kd}]$$

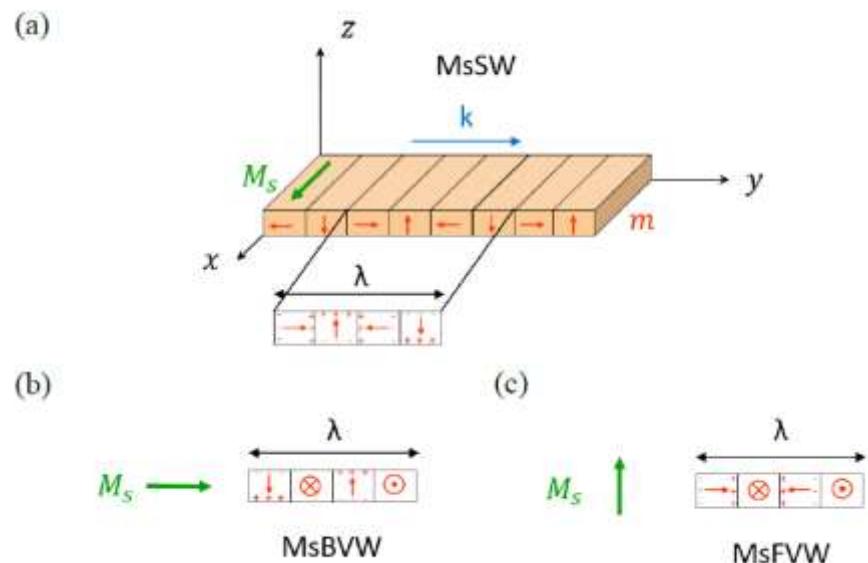
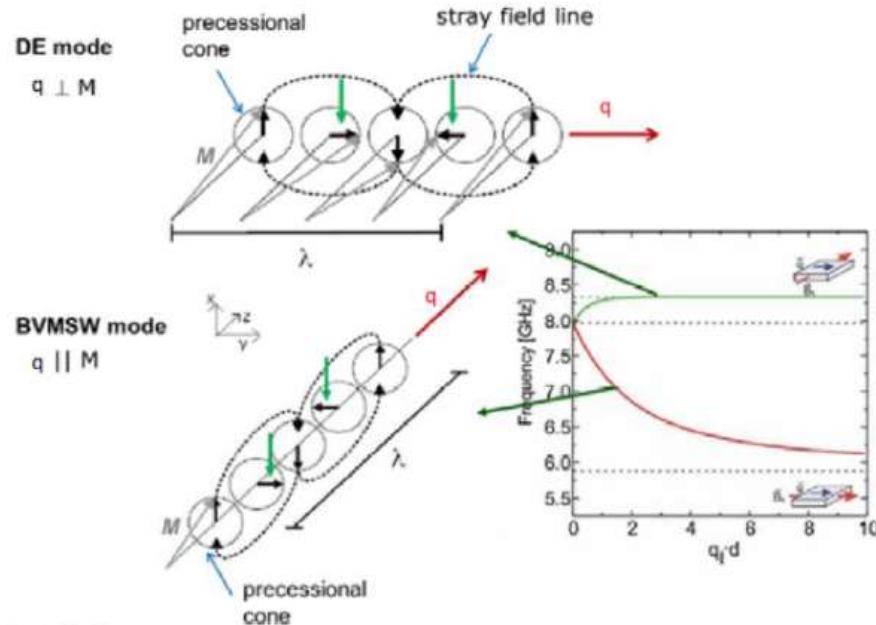
$$\left. \frac{1}{v_g} \right|_{kd=0} = \frac{4}{\omega_M d} \frac{\sqrt{\omega_0(\omega_0 + \omega_M)}}{\omega_M}$$



$$\omega_0 = \gamma \mu_0 H_0$$

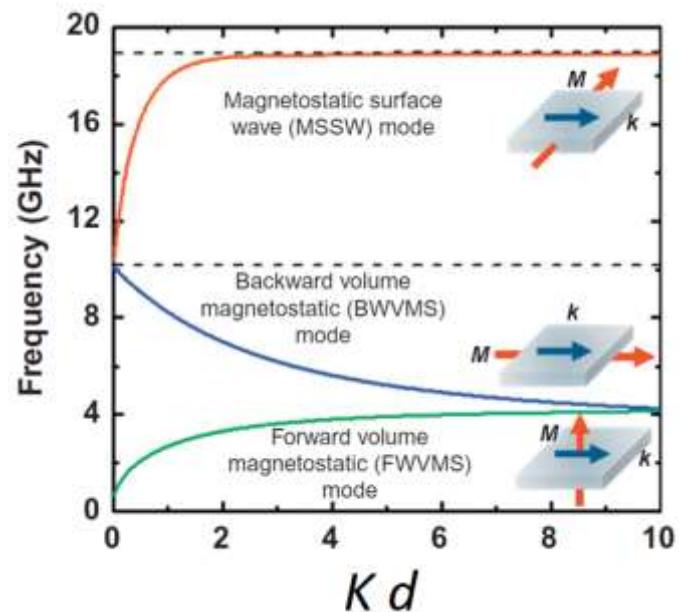
$$\omega_M = \gamma \mu_0 M_s$$

- Surface mode
- Anisotropic propagation: $k \perp M$
- Non-reciprocal propagation (on the same interface): $K = H \times n$
- Positive group velocity



Martin Collet. Dynamique d'ondes de spin dans des microstructures à base de films de YIG ultra-minces : vers des dispositifs magnoniques radiofréquences. Science des matériaux [cond-mat.mtrl-sci]. Université Paris-Saclay, 2017. Français. <NNT : 2017SACLSS39>. <tel-01721374>

Thin films : $d < 100$ nm

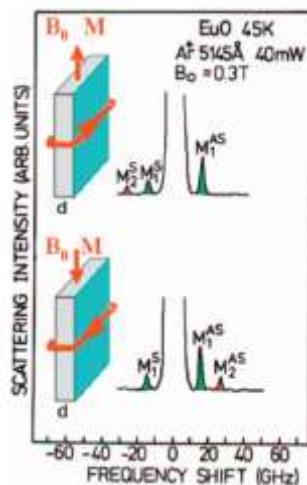
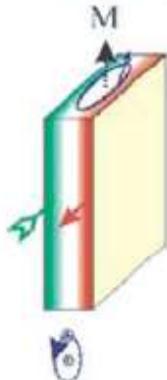
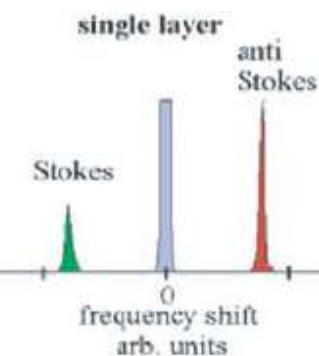


courtesy of B Hillbrands

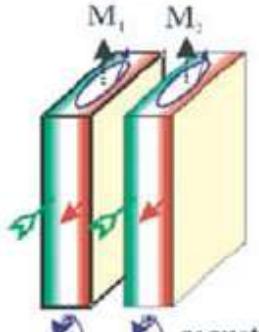
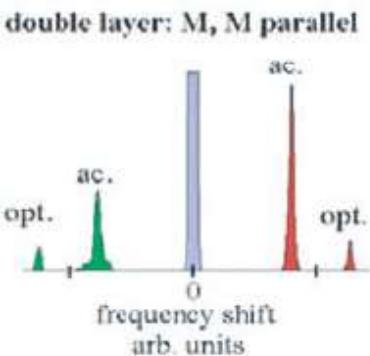
Back to the future (1988): exchange coupled layers



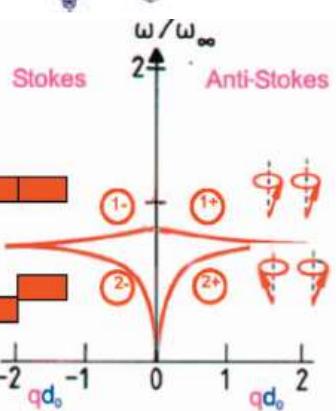
(a)



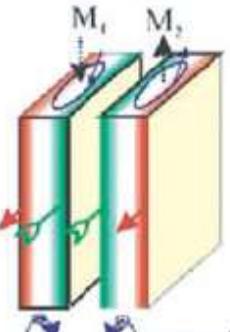
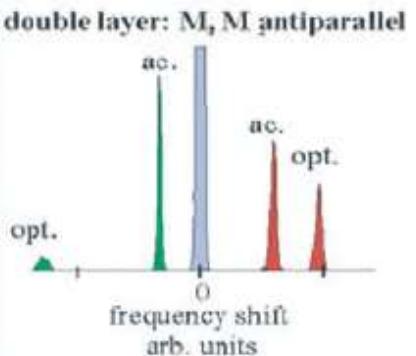
(b)



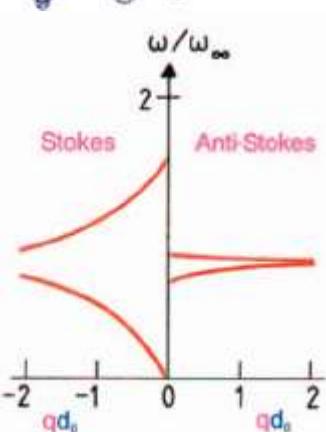
acoustic
optic



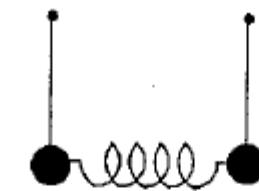
(c)



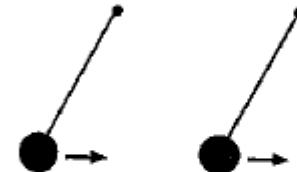
acoustic
optic



(a)



(b)

 ω_0

(c)

 $\omega_0 + \Delta$

Figure 19. (a) Two pendula coupled by a spring. The two possible motions are (b) an in-phase oscillation and (c) an out-of-phase oscillation.

CONCLUSION

(paraphrasing R. Feynman)

**«There is a lot of room...
...in the middle !»**