AIMAG School Perugia 2025

Basic magnetism

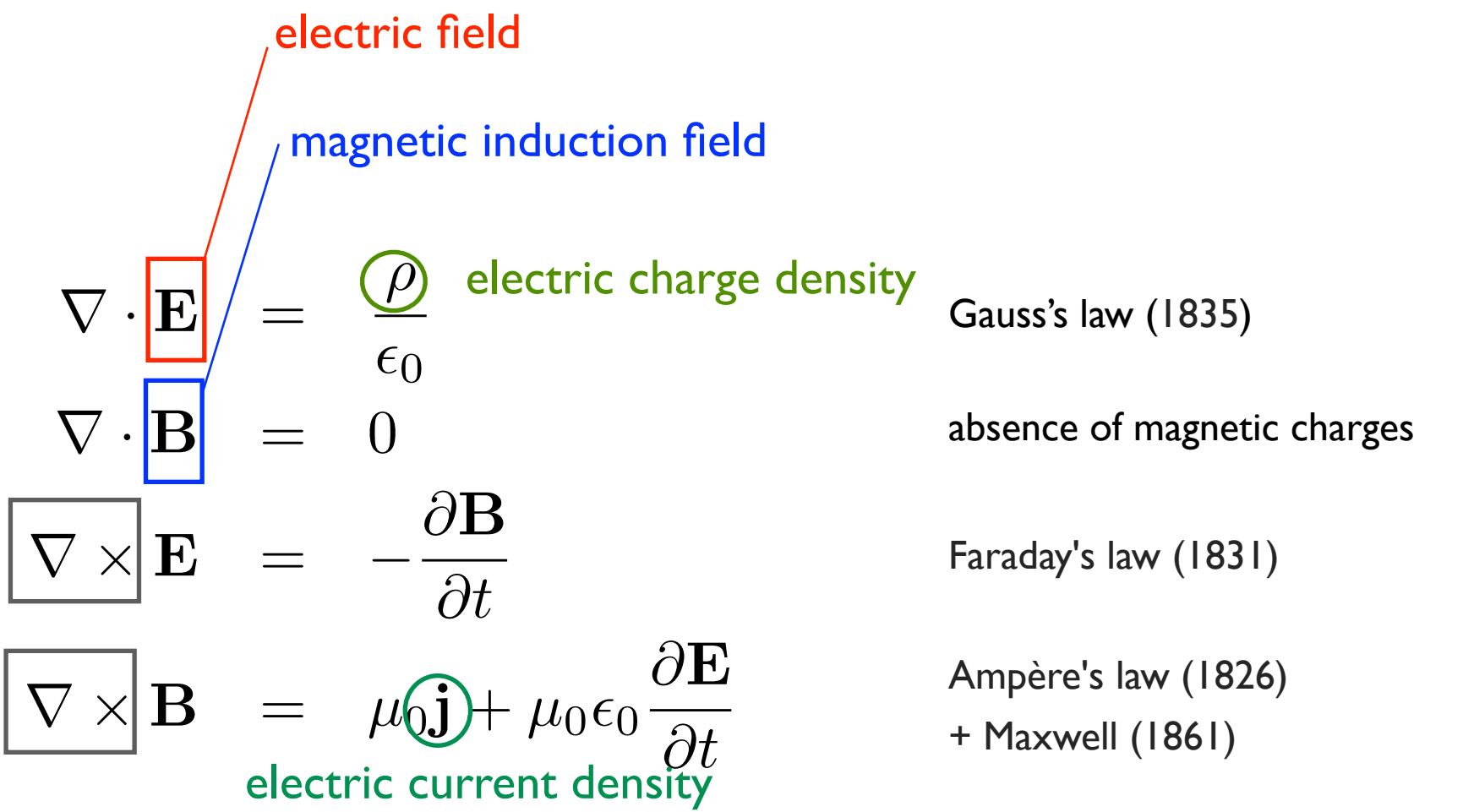
Vittorio Basso, INRIM, v.basso@inrim.it

Outline:

- **Maxwell equations with magnetic media**
- **Demagnetizing effects**

- **Ferromagnetism:**
  - **spontaneous magnetization**
  - **exchange**
  - **anisotropy**

Maxwell equations



$$\mu_0 = 4\pi[1 + 2.0(2.3) \times 10^{-10}] \times 10^{-7}\mathrm{H/m}$$

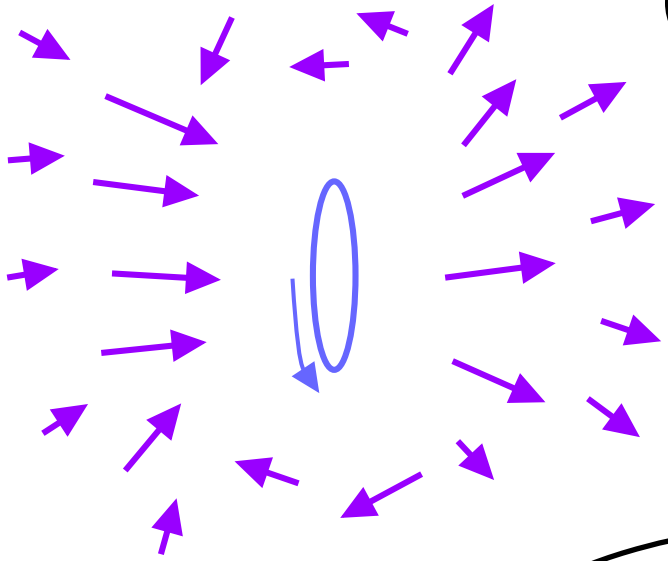
$$\epsilon_0 = 1/(c^2\mu_0) = 8.85 \cdot 10^{-12}\ \mathrm{F/m}$$

The speed of light
c = 2.99792458 × 10^8 m s^{-1}

where the place for magnetized matter?

The magnetic induction field (B) produced in space by

- a wire with electric current

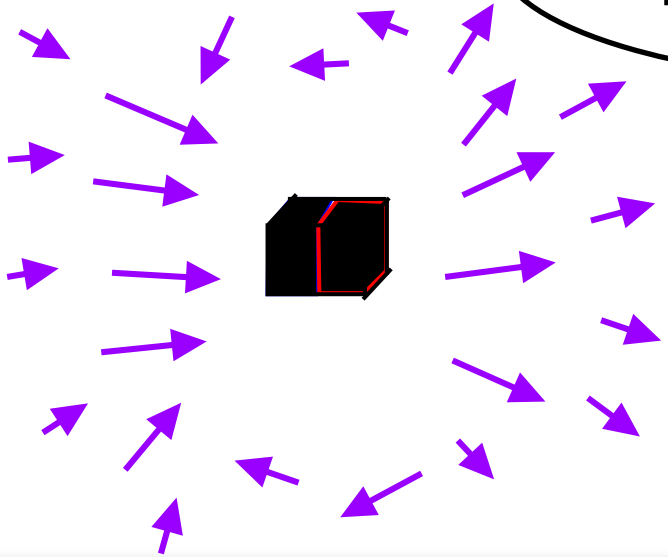


"is the same!"



H. C. Oersted (1777 - 1851)

- a permanent magnet

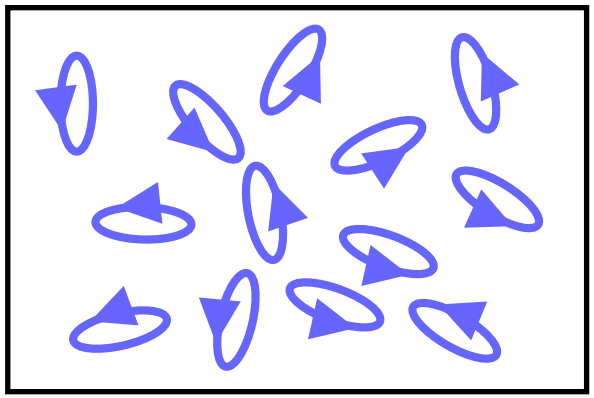


"..then
magnetic materials must be
composed by many microscopic
current loops"



A. M. Ampere (1775-1836)

Ampere hypothesis



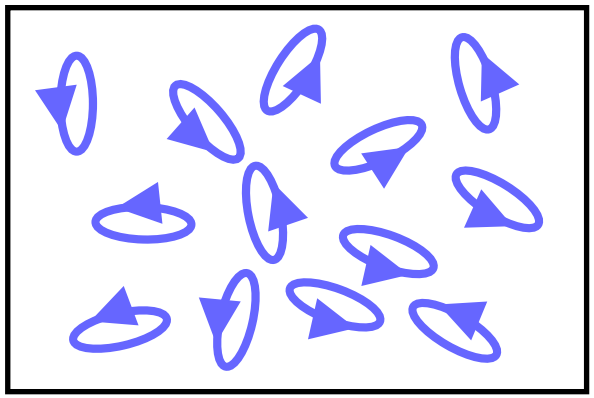
$\mathbf{j}_{micro}$



A.M.Ampere (1775-1836)

"Magnetic materials should be composed by many microscopic current loops"

Magnetized media



$\mathbf{j}_{micro}$

magnetizing currents

$$\mathbf{j}_M(\mathbf{r}) = \frac{1}{\Delta V}\int_{\Delta V} \mathbf{j}_{micro}(\mathbf{r}')d^3r'$$

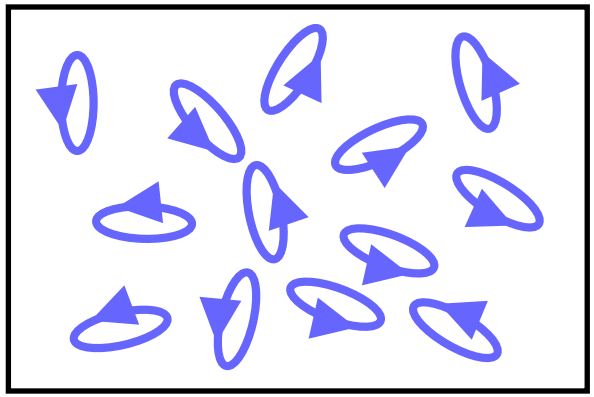
moving averages

$$F(\mathbf{r}) = \frac{1}{\Delta V}\int_{\Delta V} F_{micro}d^3r$$

Magnetized media

1) Take the moving average of the microscopic currents

$$\mathbf{j}_M(\mathbf{r}) = \frac{1}{\Delta V}\int_{\Delta V} \mathbf{j}_{micro}(\mathbf{r}')d^3r'$$



2) that are defined only inside the magnetic body

$$\nabla \cdot \mathbf{j}_M(\mathbf{r}) = 0$$

3) With zero divergence we can introduce **M**

$$\mathbf{j}_M = \nabla \times \mathbf{M}$$

Magnetized media

1) In the Maxwell equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

we introduce

$$\mathbf{j} \rightarrow \mathbf{j} + \mathbf{j}_M$$

with $\mathbf{j}_M = \nabla \times \mathbf{M}$

2) Then

$$\nabla \times (\mathbf{B} - \mu_0 \mathbf{M}) = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

or better

$$\nabla \times \mathbf{H} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mu_0 \mathbf{H} = \mathbf{B} - \mu_0 \mathbf{M}$$

magnetic field

magnetic induction

magnetization

Magnetized media

Magnetization **M**

$$\mathbf{j}_M = \nabla \times \mathbf{M}$$

magnetic dipole moment

$$\boldsymbol{\mu} = \int_\Omega \frac{\mathbf{r}' \times \mathbf{j}(r')}{2} d^3 r'$$

$$\boldsymbol{\mu} = \int_\Omega \frac{\mathbf{r}' \times (\nabla \times \mathbf{M}(r'))}{2} d^3 r' = \int_\Omega \mathbf{M}(r') d^3 r'$$

magnetic dipole moment

M is the density of magnetic moment

Maxwell equations in matter

electric displacement field

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

magnetic induction field

$$\nabla \cdot \mathbf{D} = \rho$$

ρ: electric charge density

Gauss's law (1835)

$$\nabla \cdot \mathbf{B} = 0$$

absence of magnetic charges

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's law (1831)

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampère's law (1826)
+ Maxwell (1861)

$\mathbf{j}$: electric current density

electric field

magnetic field

$$\mu_0 \mathbf{H} = \mathbf{B} - \mu_0 \mathbf{M}$$

Constitutive equations

$\mathbf{M}(\mathbf{H})$

$\mathbf{P}(\mathbf{E})$

Maxwell equations

in matter

Magnetostatics

$$\nabla \cdot \mathbf{D} = \rho$$ Gauss's law (1835)

$$\nabla \cdot \mathbf{B} = 0$$ absence of magnetic charges

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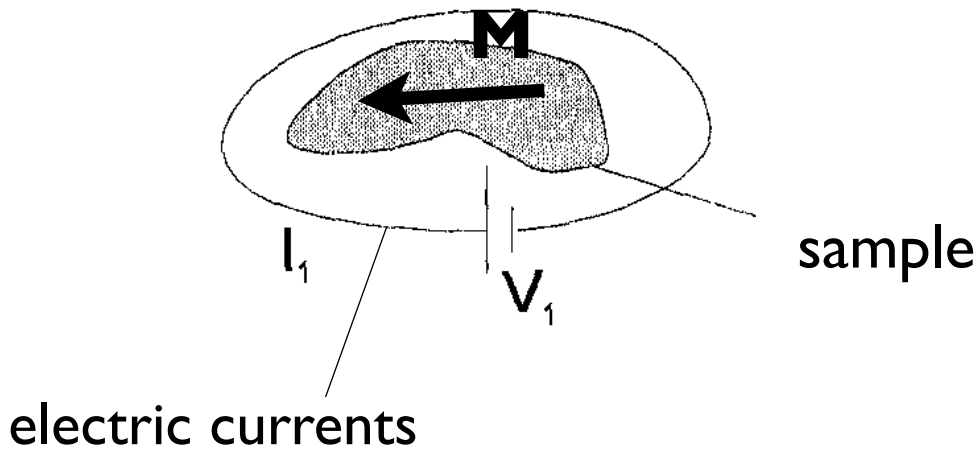
Magnetostatics

Equations of magnetostatics

$$\begin{cases} \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \\ \nabla \times \mathbf{H} = \mathbf{j} \end{cases}$$

conservative

solenoidal



Magnetostatics

Equations for the magnetostatic field

$$\begin{cases} \nabla \cdot \mathbf{H}_M &= -\nabla \cdot \mathbf{M} \\ \nabla \times \mathbf{H}_M &= 0 \end{cases}$$

conservative

Magnetostatics

Equations for the magnetostatic field

$$\begin{cases} \nabla \cdot \mathbf{H}_M &= -\nabla \cdot \mathbf{M} \\ \nabla \times \mathbf{H}_M &= 0 \end{cases}$$

conservative

1) Introduce the magnetic *scalar* potential

$$\mathbf{H}_M = -\nabla \phi_M$$

2) Obtain a Poisson equation for the scalar potential

$$\nabla^2 \phi_M = -\rho_M$$

The divergence of **M** takes the role of a charge density

$$\rho_M = -\nabla \cdot \mathbf{M}$$

3) Obtain a formal solution

$$\phi_M = -\frac{1}{4\pi} \int_V \frac{\nabla \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 r' + \frac{1}{4\pi} \int_\Sigma \frac{\mathbf{n} \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2 r'$$

Magnetostatics

1) The solution

$$\phi_M = -\frac{1}{4\pi}\int_V \frac{\nabla\cdot\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r' + \frac{1}{4\pi}\int_\Sigma \frac{\mathbf{n}\cdot\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^2r'$$

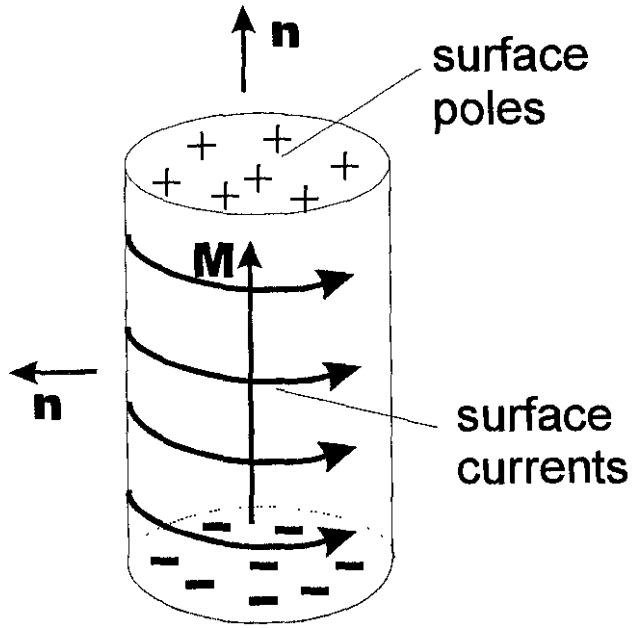
contains contributions form the volume and from the surface

$$\mathbf{H}_M = -\nabla\phi_M$$

2) Both are extremely important in magnetic materials

volume stray fields

surface demagnetizing field



Demagnetizing field

with uniform magnetization

$$\phi_M = \frac{1}{4\pi}\mathbf{M} \cdot \int_\Sigma \frac{\mathbf{n}}{|\mathbf{r}-\mathbf{r}'|} d^2 r'$$

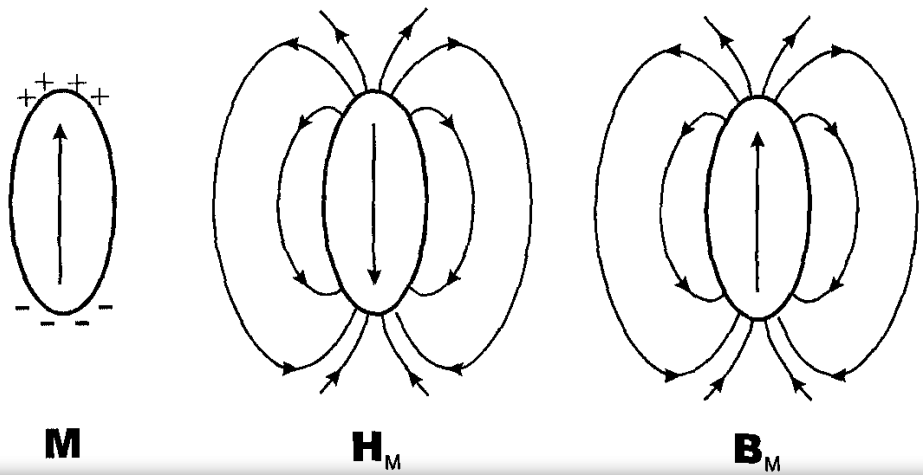
the magnetostatic field

$$\mathbf{H}_M = -\nabla \phi_M$$

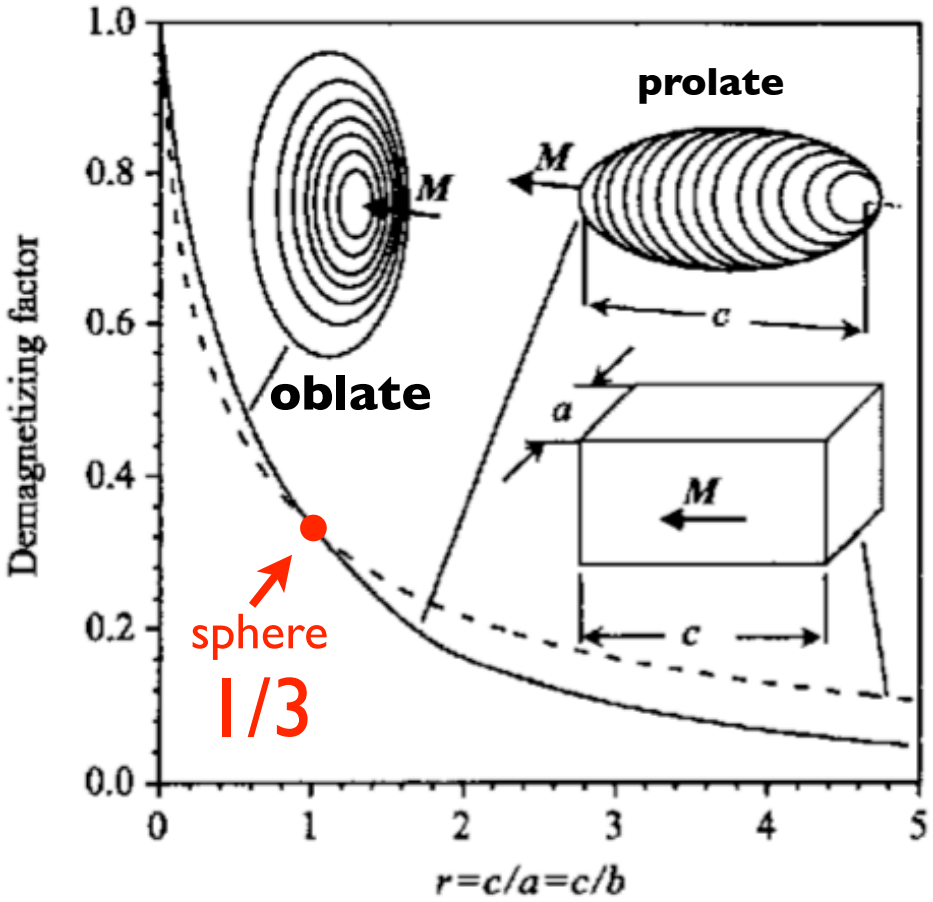
is **uniform** inside the body for *ellipsoids*!

$$\mathbf{H}_M = -N\mathbf{M}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$



Demagnetizing factor

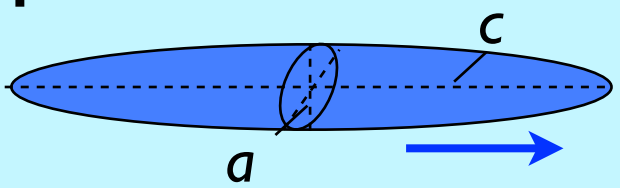


$$N_a + N_b + N_c = 1$$

Demagnetizing coefficient

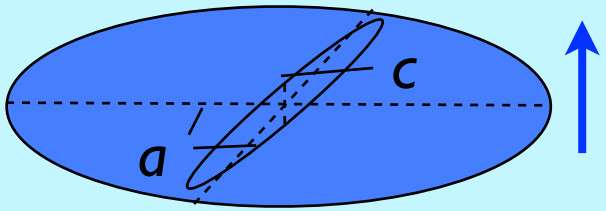
$$\mu_0 H = -N_d J$$

prolate $r = c/a$



$$N_{d_c} = \frac{1}{r^2-1}\left[\frac{r}{\sqrt{r^2-1}}\ln\left(r+\sqrt{r^2-1}\right)-1\right]$$

oblate



$$N_{d_c} = \frac{1}{1-r^2}\left[1-\frac{r}{\sqrt{1-r^2}}\arcsin\left(\sqrt{1-r^2}\right)\right]$$

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electric field

magnetic field

$$\mu_0 \mathbf{H} = \mathbf{B} - \mu_0 \mathbf{M}$$

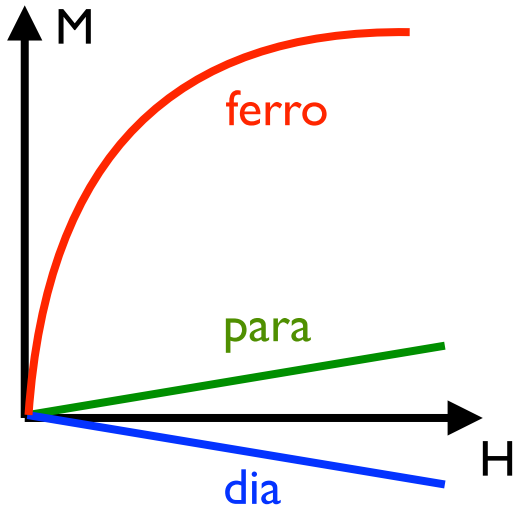
Constitutive equations

$\mathbf{M}(\mathbf{H})$

$\mathbf{P}(\mathbf{E})$

Constitutive equations

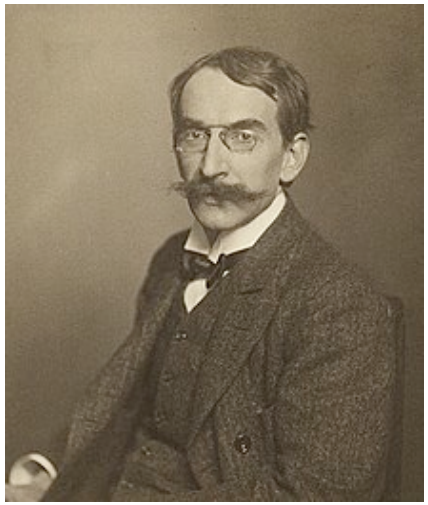
$$M(H)$$



Ferromagnets are characterized by a particularly high susceptibility.

Weiss: ferromagnets have saturated regions (magnetic domains) oriented at random and the magnetic field has only to orient the regions (and not the single atomic moments).

Domains were later observed experimentally, confirming the Weiss hypothesis.



Pierre-Ernest Weiss 1865 - 1940

Ferromagnetism

What is the origin of the forces keeping the moments parallel?

What is the origin of the atomic magnetic moment?

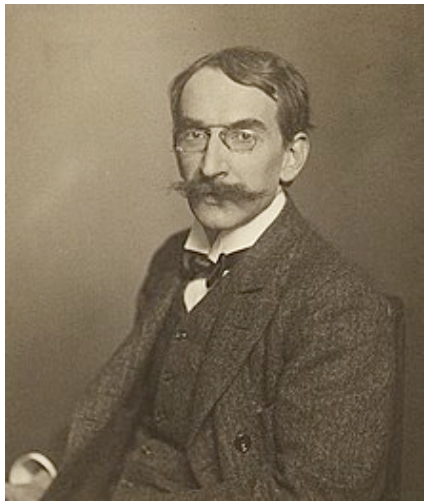
Ferromagnetism

What is the origin of the forces keeping the moments parallel?

Forces at the molecular scale keeps the elementary moment parallel

Weiss molecular field theory

$$\frac{U_W}{V} = -\frac{\mu_0}{2} N_W M^2$$



Pierre-Ernest Weiss 1865 - 1940

What is the origin of the atomic magnetic moment?

Ferromagnetism

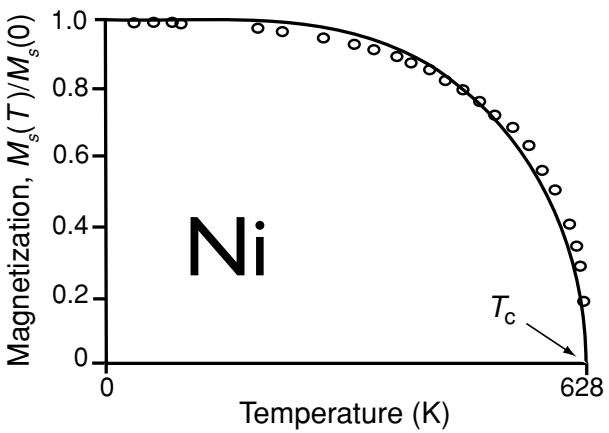
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The spontaneous order is lost at the Curie temperature T_c where $M_s = 0$





Pierre Curie 1859 - 1906

What is the origin of the atomic magnetic moment?

Ferromagnetism

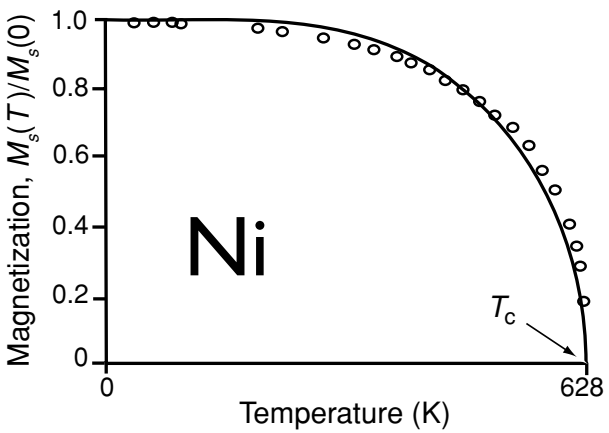
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What is the origin of the atomic magnetic moment?

Spin of electron

Quantum mechanics

Magnetism of atoms

Atomic electrons are characterized by:

- **n**, the principal quantum number (n = 1,2,3,.. is the shell number)
- **l**, the ***orbital* angular momentum** quantum number
  from 0 to n-1 (0 = s; 1 = p; 2 = d; 3 = f)
- $\mathbf{m_l}$, the magnetic quantum number (2l+1 values between -l and l)
- **s**, the ***spin* angular momentum** quantum number
  s=1/2 for electrons
- $\mathbf{m_s}$, the spin quantum number +1/2 or -1/2

Fe: $1s^2$
$2s^2\ 2p^6$
$3s^2\ 3p^6\ 3d^6$
$4s^2$

Aufbau principle (Bohr)

angular momentum → **magnetic moment**

Magnetism of atoms

Gyromagnetic ratio

$\boldsymbol{\mu = \gamma L}$

orbital

$$\gamma = -\frac{e}{2m_e}$$

$\mu = \mu_B \, m_l$

Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e}$$

$\mu_B = 9.27 \cdot 10^{-24}$ Am^2

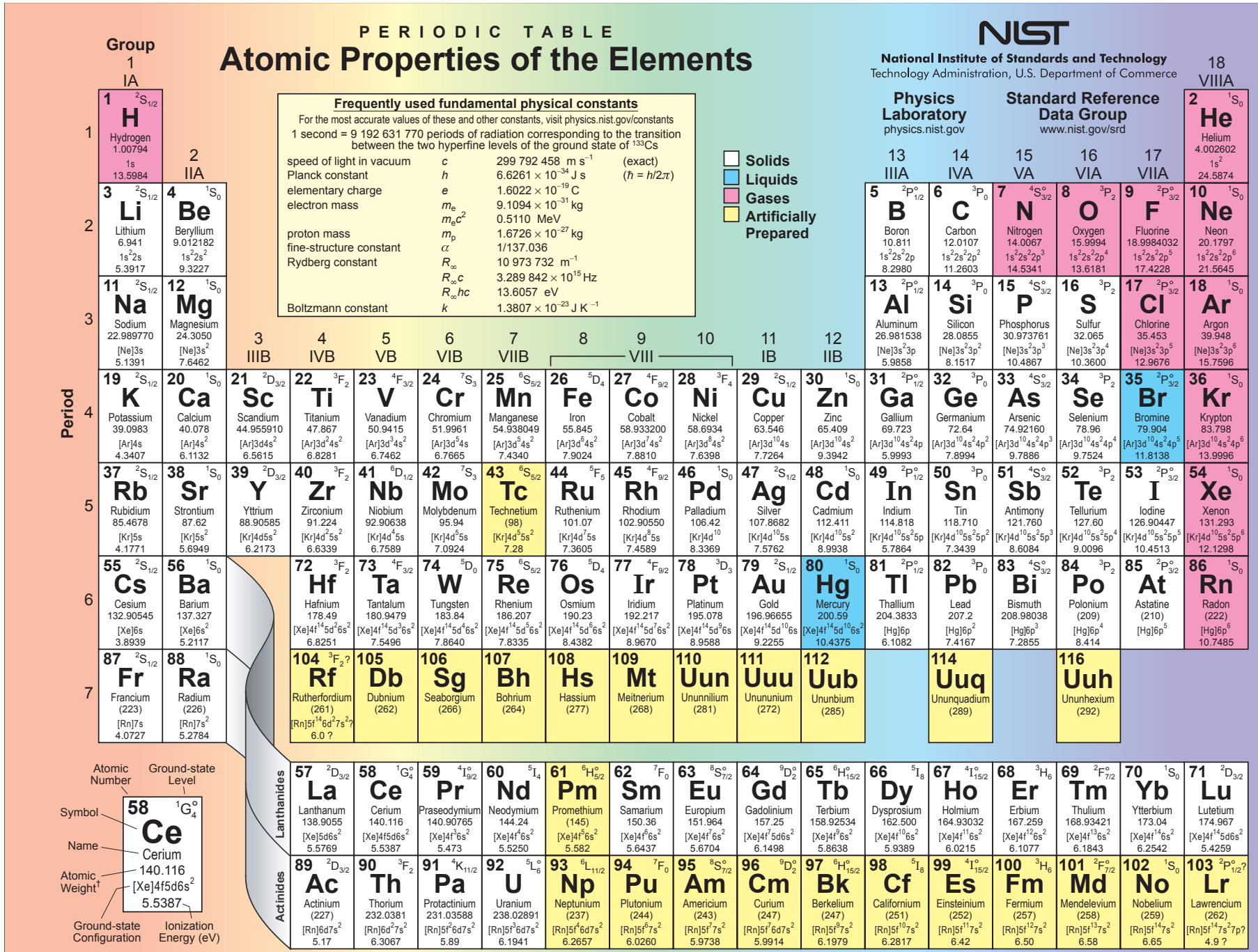
$\boldsymbol{\mu = \gamma_e S}$

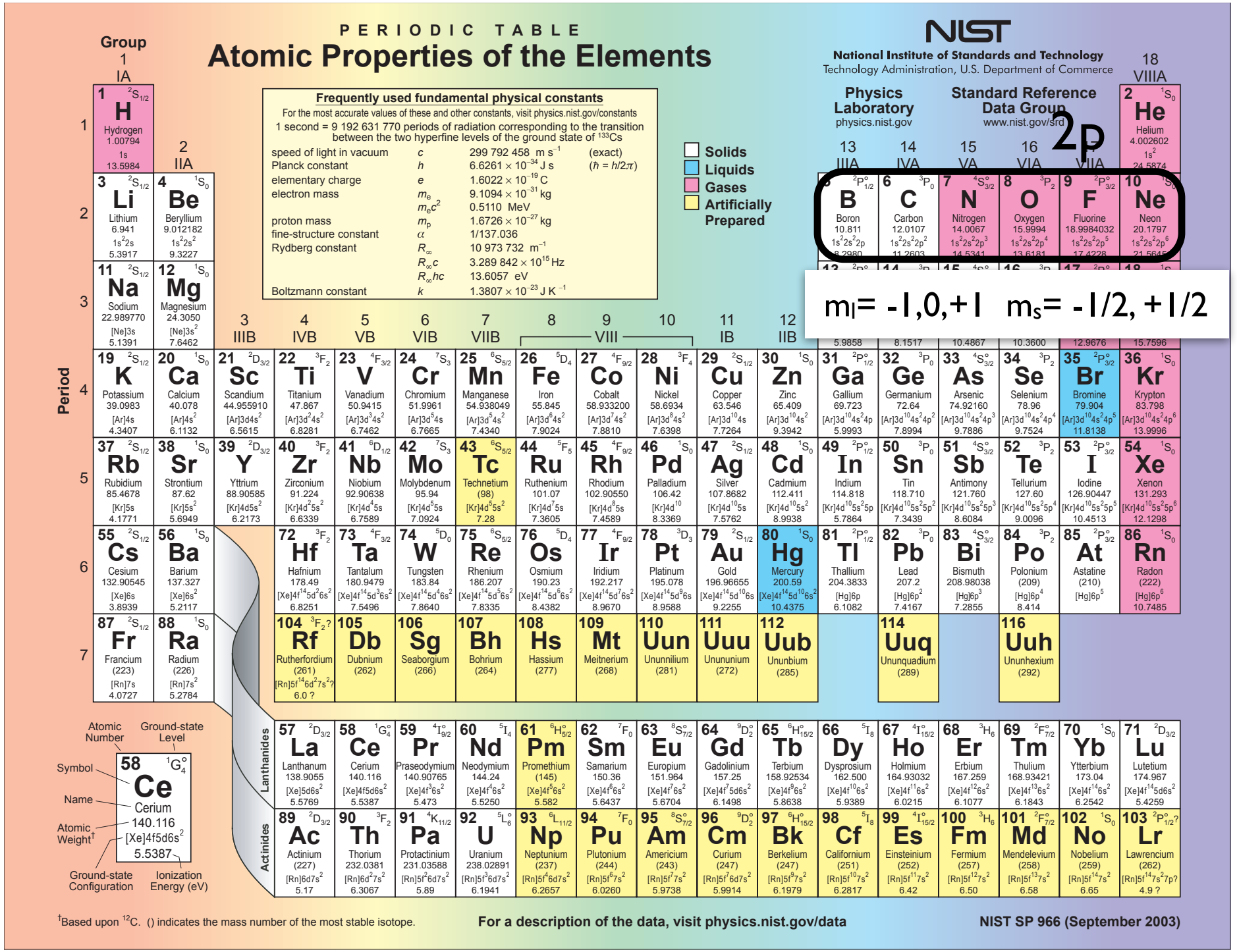
electron spin

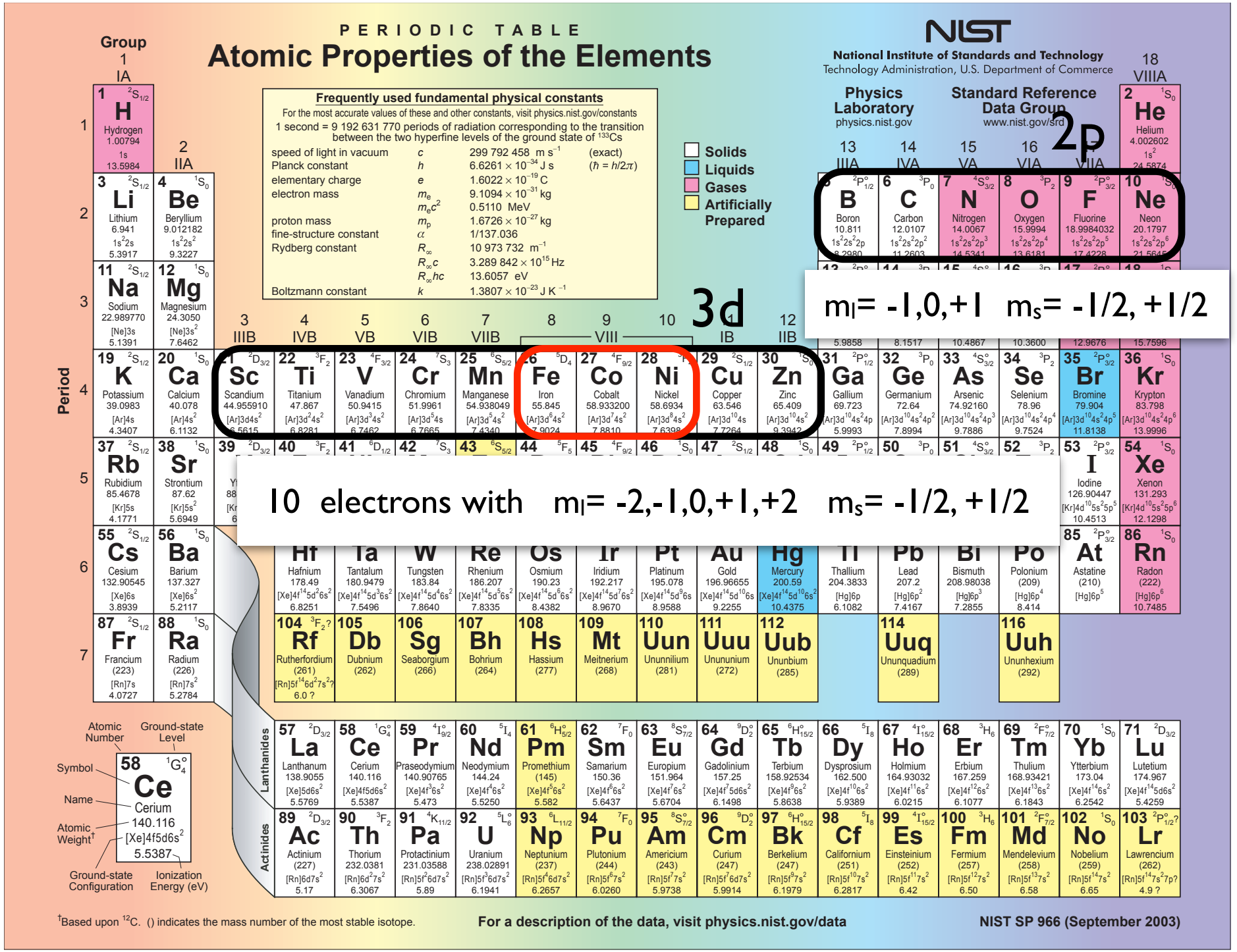
$$\gamma_e = -\frac{g_e e}{2m_e}$$

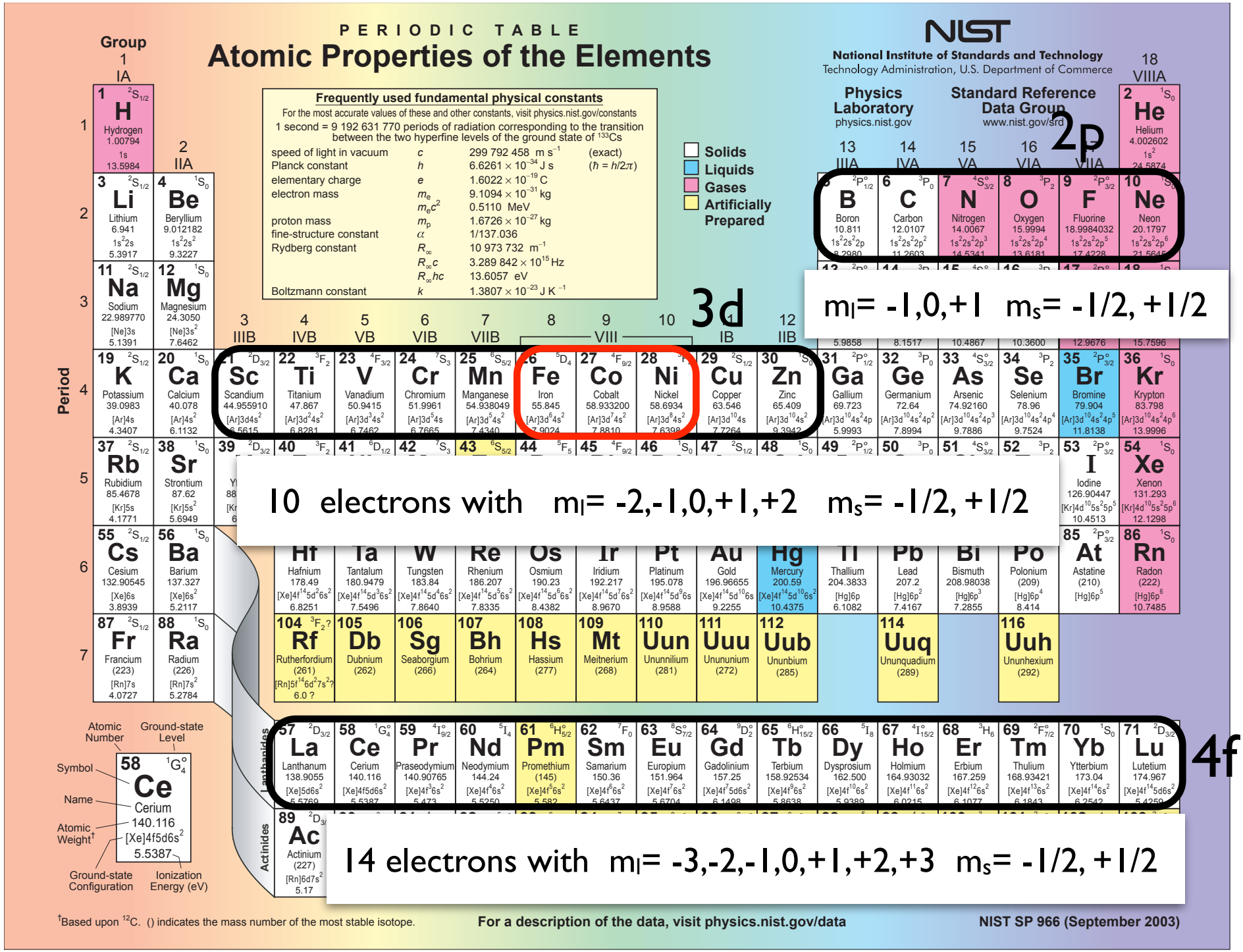
$\mu = \mu_B \, 2m_s$

$$g_e = 2\left(1 + \frac{\alpha}{2\pi} + \ldots\right)$$


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NIST
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1 IA
2 IIA
3 IIIB
4 IVB
5 VB
6 VIB
7 VIIB
8 9 10 VIII
11 IB
12 IIB
13 IIIA
14 IVA
15 VA
16 VIA
17 VIIA
18 VIIIA
Period
Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of 133Cs
speed of light in vacuum c 299 792 458 m s−1 (exact)
Planck constant h 6.6261 × 10−34 J s (ħ = h/2π)
elementary charge e 1.6022 × 10−19 C
electron mass me 9.1094 × 10−31 kg
mec2 0.5110 MeV
proton mass mp 1.6726 × 10−27 kg
fine-structure constant α 1/137.036
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NIST SP 966 (September 2003)


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14 electrons with mI= -3,-2,-1,0,+1,+2,+3 ms= -1/2, +1/2
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For a description of the data, visit physics.nist.gov/data
NIST SP 966 (September 2003)

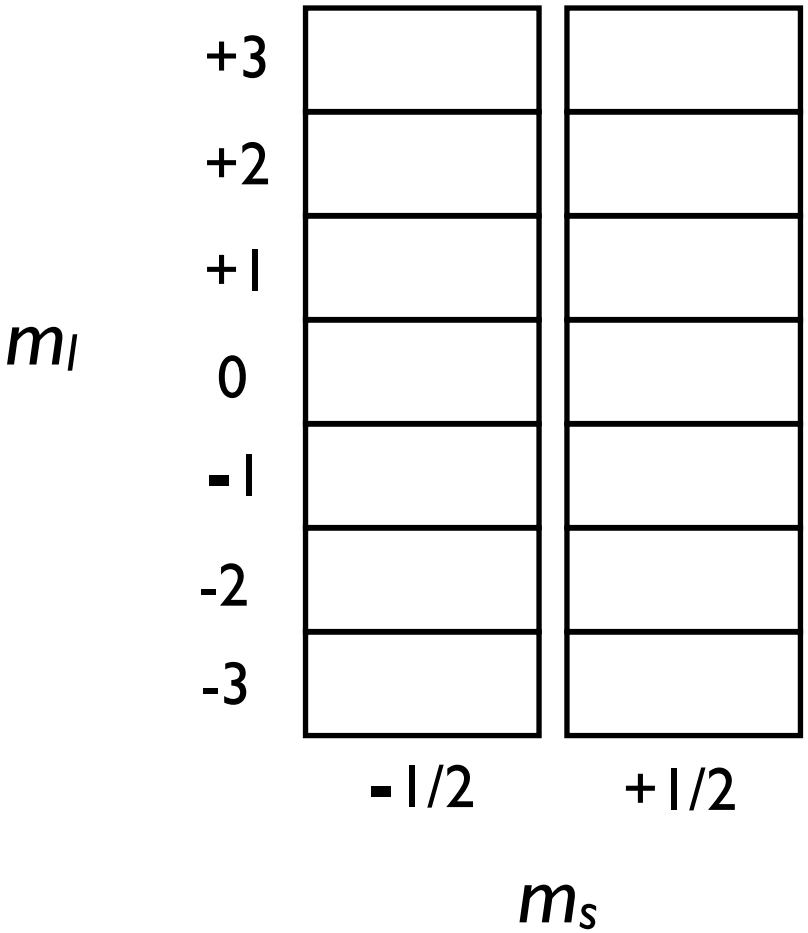
Magnetism of atoms

Hund's rules:

rule (i) The electrons occupies states in order to give the maximum total S

rule (ii) The electrons occupies states in order to give the maximum total L

rule (iii) S and L combine to give J as J= L−S if the shell less then half occupied and J= L+ S if the shell is more then half occupied.



spin orbit energy **L·S**

4f electrons

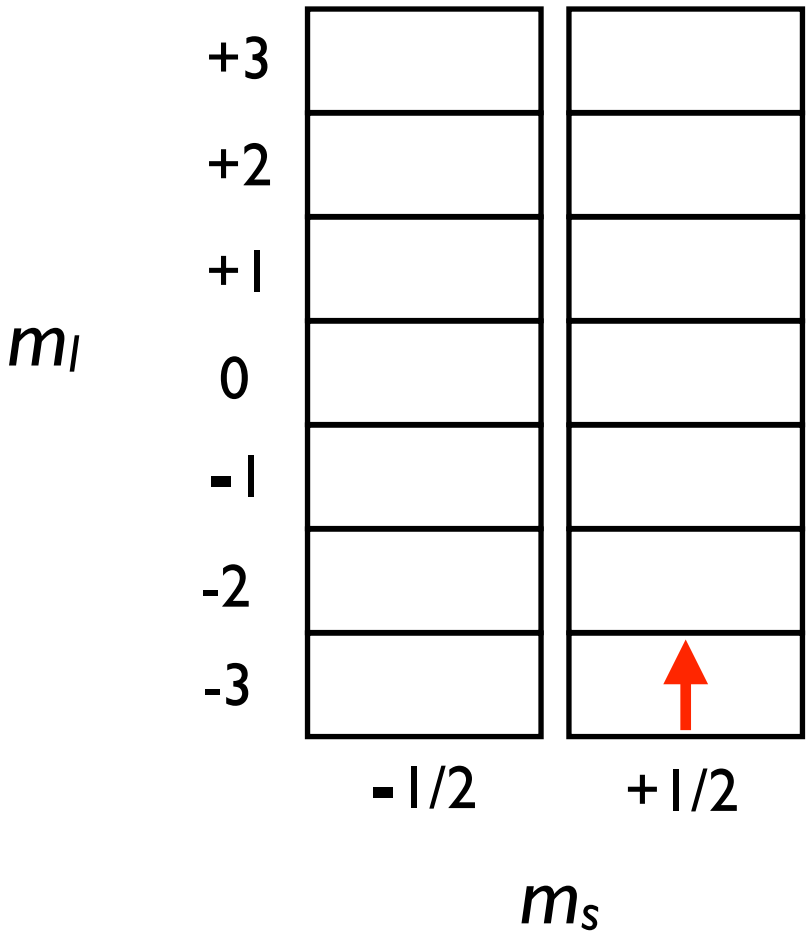
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spin orbit energy **L·S**

4f electrons

Ce^{+3} $4f^{1}$

S = 1/2

L = 3

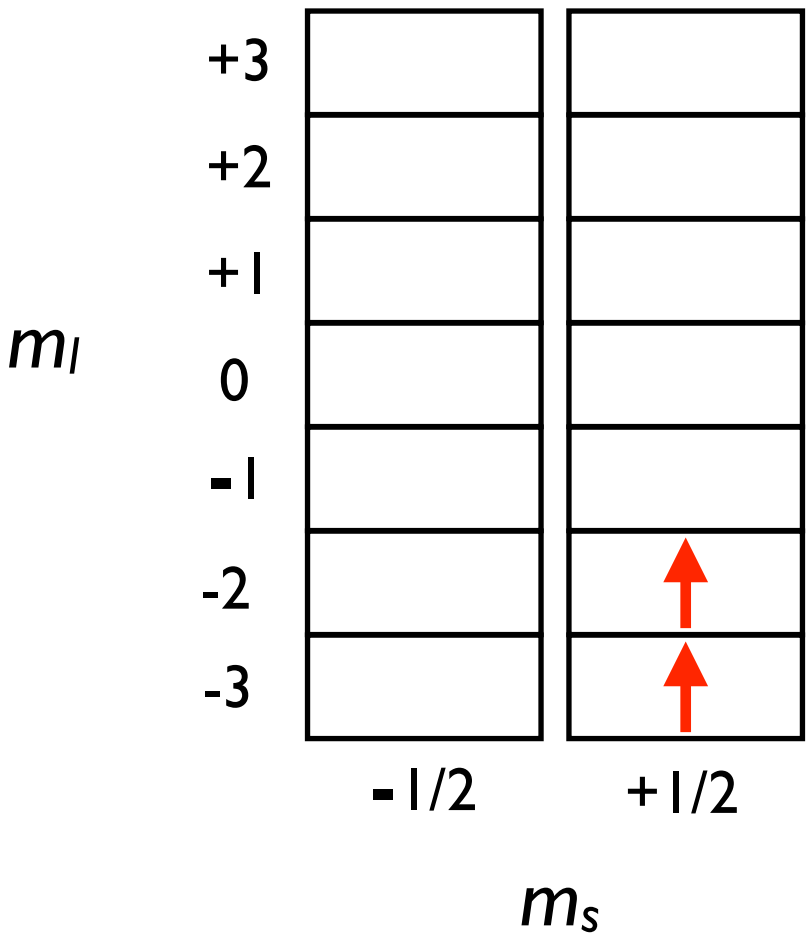
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spin orbit energy **L·S**

4f electrons

Pr^{+3} $4f^{2}$

S = 2/2

L = 5

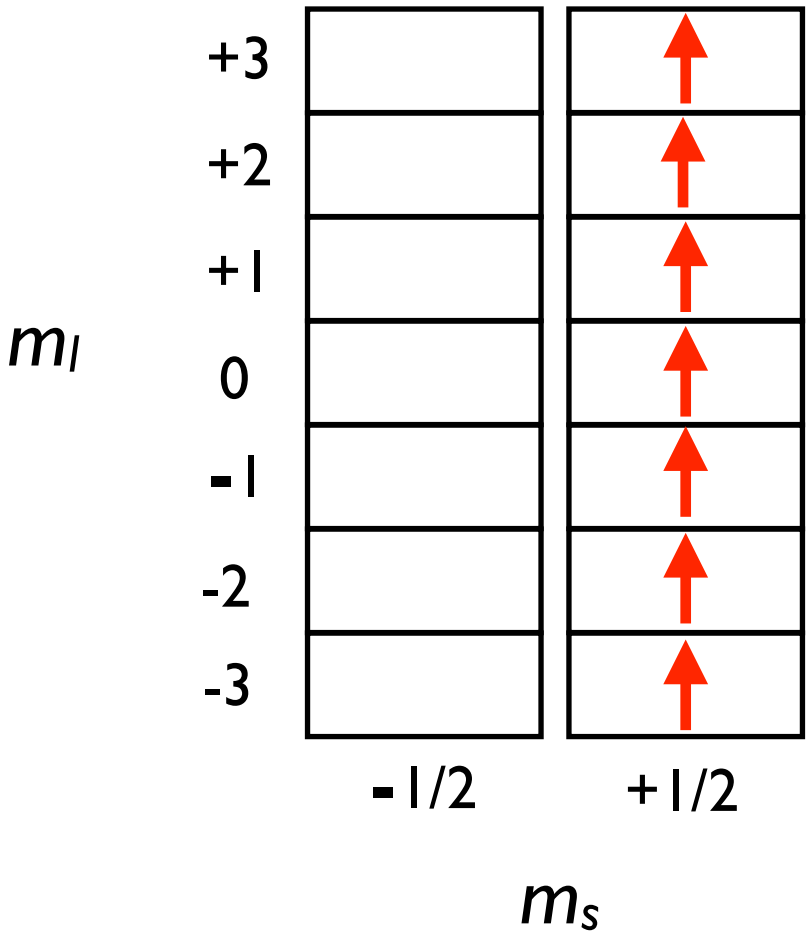
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spin orbit energy **L·S**

4f electrons

Gd^{+3} $4f^{7}$

S = 7/2

L = 0

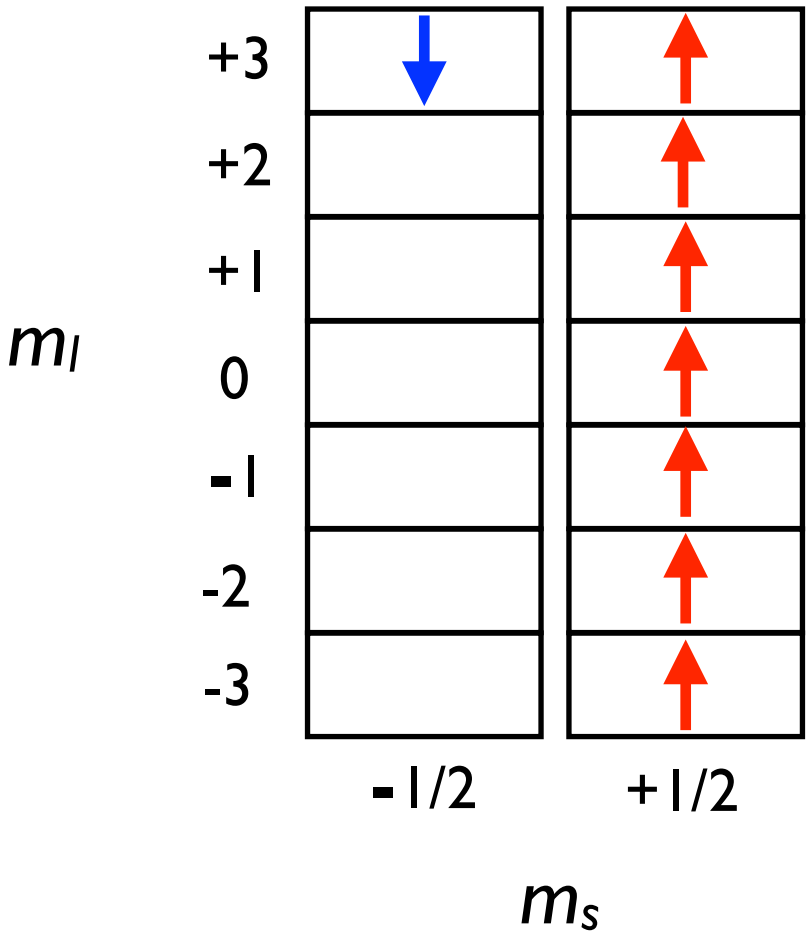
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spin orbit energy **L·S**

4f electrons

Tb^{+3} $4f^{7}$

S = 6/2

L = 3

Magnetism of atoms

Hund's rules:

rule (i) The electrons occupies states in order to give the maximum total S

rule (ii) The electrons occupies states in order to give the maximum total L

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4f electrons

Element	z	e configuration	4f of $^{+3}$ ion	S	L	J	g_J	$g_J J$
Lanthanum (La)	57	[Xe] $4f^0\ 5d^1\ 6s^2$	0	0	0	0	-	0.00
Cerium (Ce)	58	[Xe] $4f^1\ 5d^1\ 6s^2$	1	1/2	3	5/2	6/7	2.14
Praseodymium (Pr)	59	[Xe] $4f^3\ 5d^0\ 6s^2$	2	2/2	5	8/2	4/5	3.20
Neodymium (Nd)	60	[Xe] $4f^4\ 5d^0\ 6s^2$	3	3/2	6	9/2	8/11	3.27
Promethium (Pm)	61	[Xe] $4f^5\ 5d^0\ 6s^2$	4	4/2	6	8/2	3/5	2.40
Samarium (Sm)	62	[Xe] $4f^6\ 5d^0\ 6s^2$	5	5/2	5	5/2	2/7	0.71
Europium (Eu)	63	[Xe] $4f^7\ 5d^0\ 6s^2$	6	6/2	3	0	3/2	0.00
Gadolinium (Gd)	64	[Xe] $4f^7\ 5d^1\ 6s^2$	7	7/2	0	7/2	2	7.00
Terbium (Tb)	65	[Xe] $4f^9\ 5d^0\ 6s^2$	8	6/2	3	12/2	3/2	9.00
Dysprosium (Dy)	66	[Xe] $4f^{10}\ 5d^0\ 6s^2$	9	5/2	5	15/2	4/3	10.00
Holmium (Ho)	67	[Xe] $4f^{11}\ 5d^0\ 6s^2$	10	4/2	6	16/2	5/4	10.00
Erbium (Er)	68	[Xe] $4f^{12}\ 5d^0\ 6s^2$	11	3/2	6	15/2	6/5	9.00
Thulium (Tm)	69	[Xe] $4f^{13}\ 5d^0\ 6s^2$	12	2/2	5	12/2	7/6	7.00
Ytterbium (Yb)	70	[Xe] $4f^{14}\ 5d^0\ 6s^2$	13	1/2	3	7/2	8/7	4.00
Lutetium (Lu)	71	[Xe] $4f^{14}\ 5d^1\ 6s^2$	14	0	0	0	-	0.00

Magnetism of 3d metals

3d electrons

n of 3d electrons	ions	S	L	J	g_J	$g_J J$
0	Sc^{3+}	0	0	0	-	0.00
1	Ti^{3+}, V^{4+}	1/2	2	3/2	4/5	1.20
2	V^{3+}, Ti^{2+}	2/2	3	4/2	2/3	1.33
3	Cr^{3+}, V^{2+}	3/2	3	3/2	2/5	0.60
4	Mn^{3+}, Cr^{2+}	4/2	2	0	-	0.00
5	Fe^{3+}, Mn^{2+}	5/2	0	5/2	2	5.00
6	Co^{3+}, Fe^{2+}	4/2	2	8/2	3/2	6.00
7	Ni^{3+}, Co^{2+}	3/2	3	9/2	4/3	6.00
8	Cu^{3+}, Ni^{2+}	2/2	3	8/2	5/4	5.00
9	Zn^{3+}, Cu^{2+}	1/2	2	5/2	6/5	3.00
10	Ga^{3+}, Zn^{2+}	0	0	0	-	0.00

Magnetism of 3d metals

3d electrons

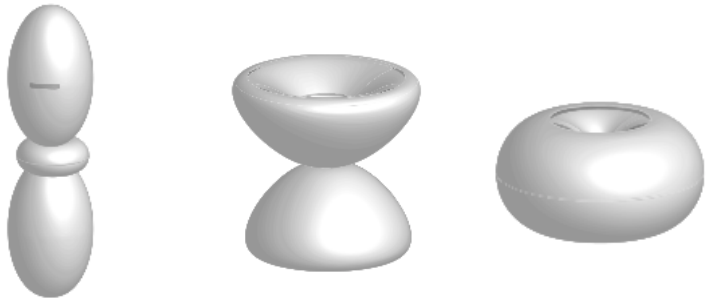
n of 3d electrons	ions	S	L	J	g_J	$g_J J$
0	Sc^{3+}	0	0	0	-	0.00
1	Ti^{3+}, V^{4+}	1/2	2	3/2	4/5	1.20
2	V^{3+}, Ti^{2+}	2/2	3	4/2	2/3	1.33
3	Cr^{3+}, V^{2+}	3/2	3	3/2	2/5	0.60
4	Mn^{3+}, Cr^{2+}	4/2	2	0	-	0.00
5	Fe^{3+}, Mn^{2+}	5/2	0	5/2	2	5.00
6	Co^{3+}, Fe^{2+}	4/2	2	8/2	3/2	6.00
7	Ni^{3+}, Co^{2+}	3/2	3	9/2	4/3	6.00
8	Cu^{3+}, Ni^{2+}	2/2	3	8/2	5/4	5.00
9	Zn^{3+}, Cu^{2+}	1/2	2	5/2	6/5	3.00
10	Ga^{3+}, Zn^{2+}	0	0	0	-	0.00

	$\mu_0 M_s$(4 K) (T)	δ (kg m^{-3})	m (kg mol^{-1})	n_e
Fe	2.22	7874	0.0558	2.24
Co	1.85	8900	0.0589	1.75
Ni	0.64	8908	0.0587	0.60

Non integer values!!

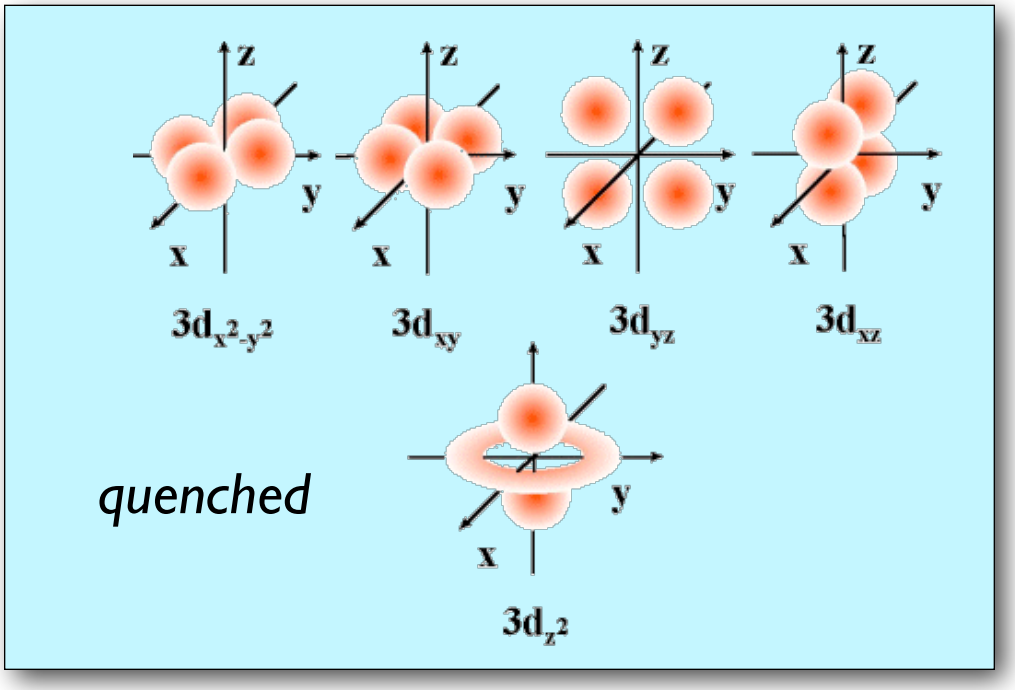
Atomic orbitals

unquenched



$m_l=0$ $m_l=\pm1$ $m_l=\pm2$

quenched



For 4f almost **unquenched**

For 3d almost **quenched**

- *almost only* the spin matters

Exchange energy

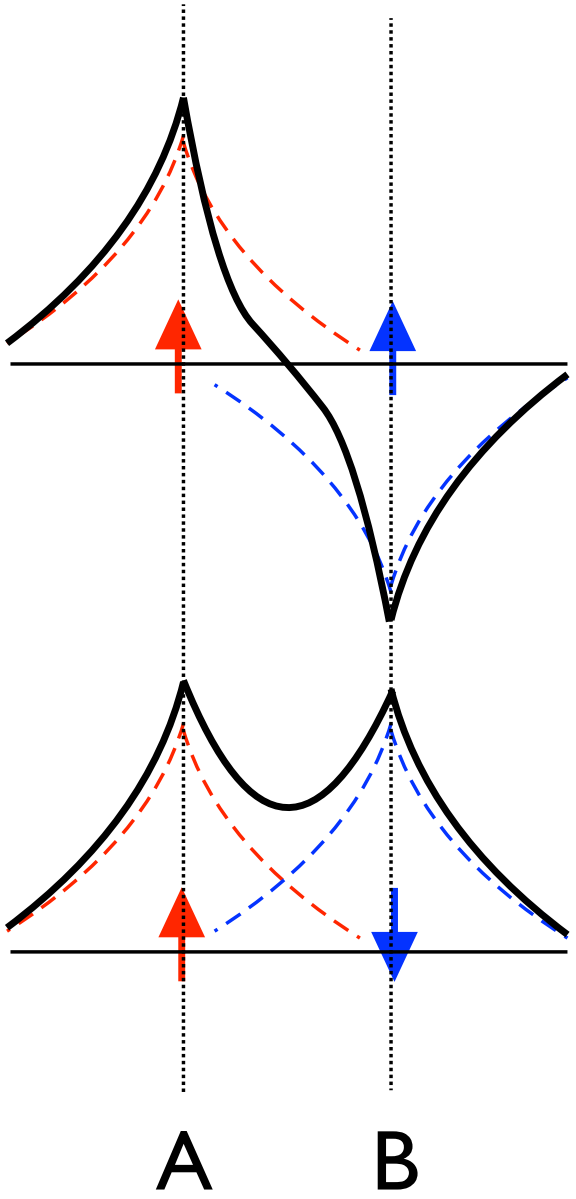
Electronic wave functions must be antisymmetric under **exchange**!

$$\psi(r_1,r_2) = -\psi(r_2,r_1)$$

Exchange energy

Electronic wave functions must be antisymmetric under **exchange**!

$$\psi(r_1,r_2) = -\psi(r_2,r_1)$$



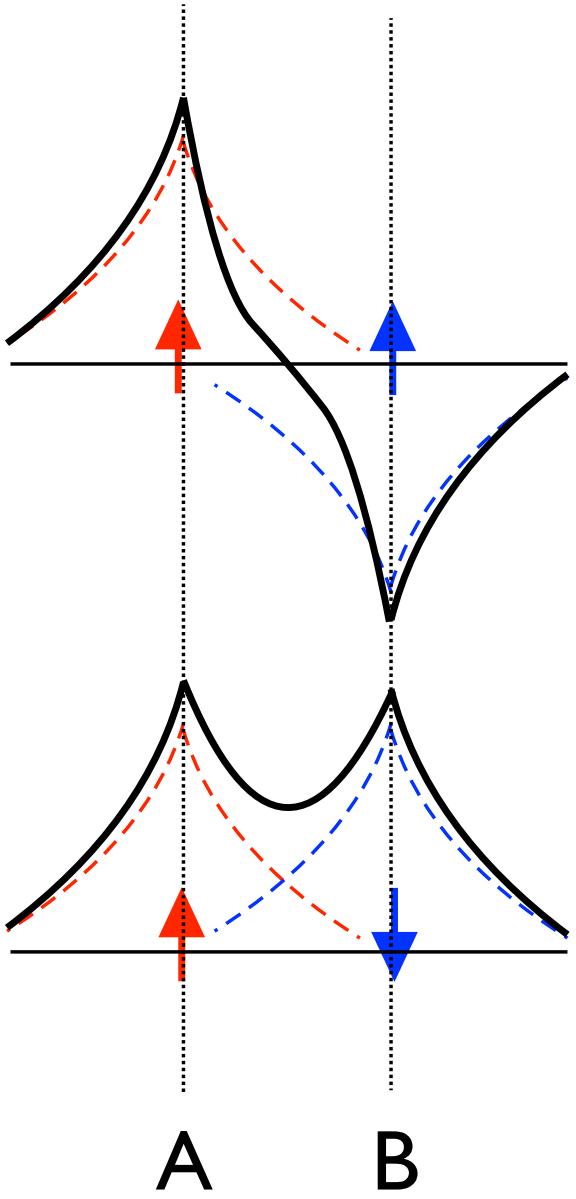
antibonding

bonding

Exchange energy

Electronic wave functions must be antisymmetric under **exchange**!

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antibonding

bonding

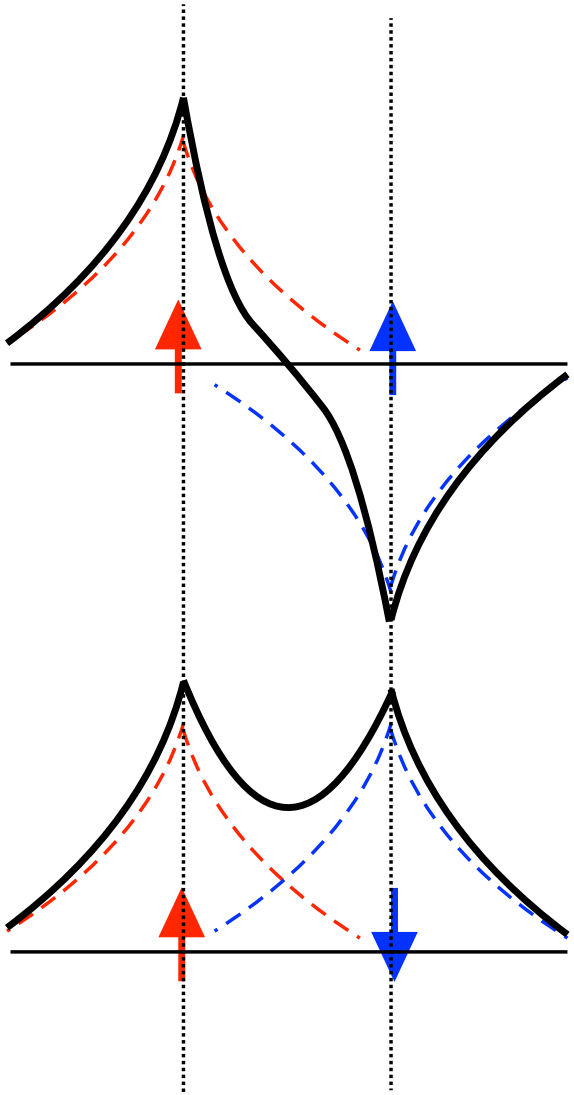
H_2 molecule

low energy state

Exchange energy

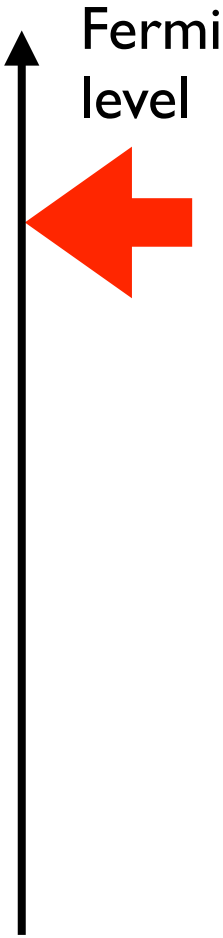
Electronic wave functions must be antisymmetric under **exchange**!

$$\psi(r_1,r_2) = -\psi(r_2,r_1)$$



antibonding high energy

bonding low energy



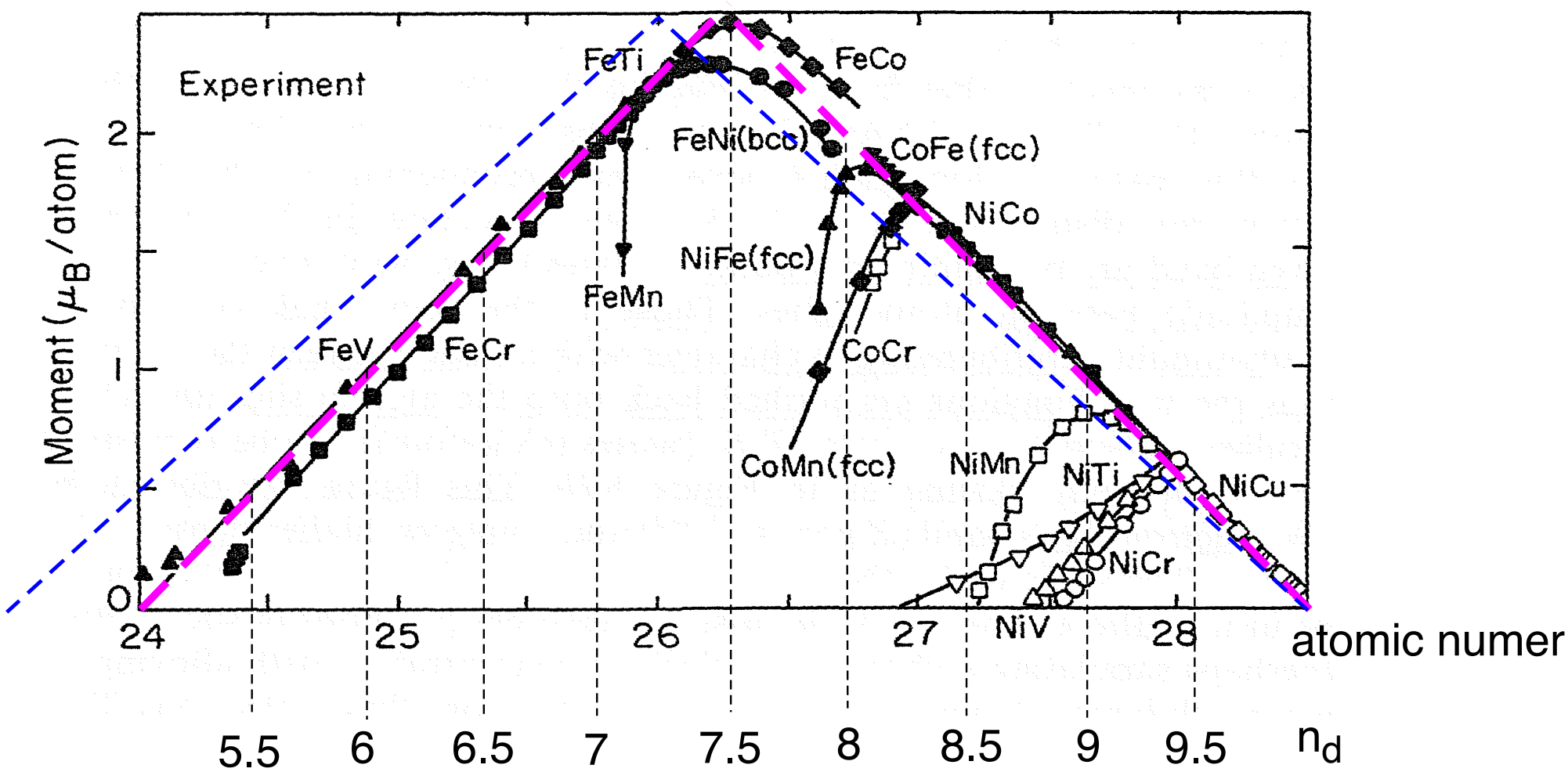
solids

A B

antibonding electrons contribute to magnetic moment !

Slater-Pauling curve

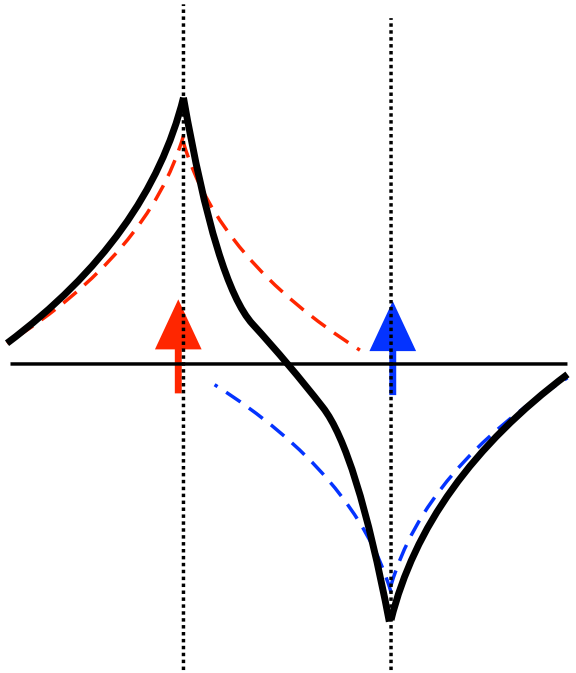
magnetic moment per atom (3d)



antibonding electrons contribute to magnetic moment !

Exchange energy

Electronic wave functions must be antisymmetric under **exchange**!



$$\psi(r_1,r_2) = -\psi(r_2,r_1)$$

antibonding

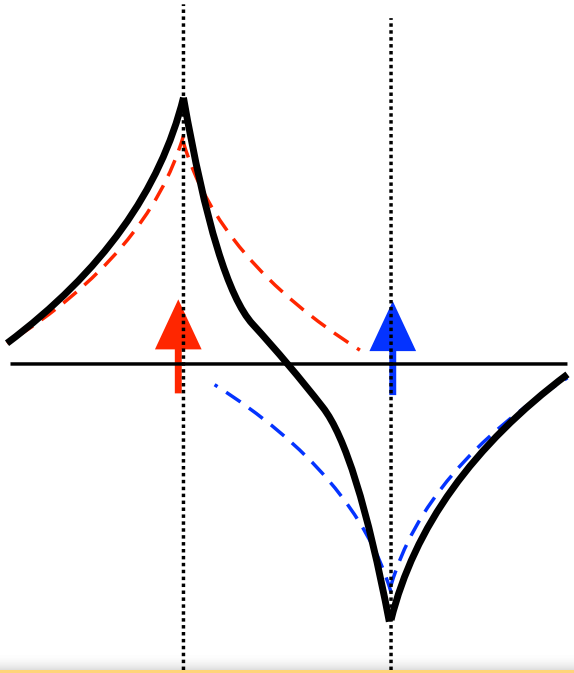
Heisenberg model

$$-J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$$

$J_{ij} > 0$ Coulomb interaction between electrons

Exchange energy

Electronic wave functions must be antisymmetric under **exchange**!



antibonding

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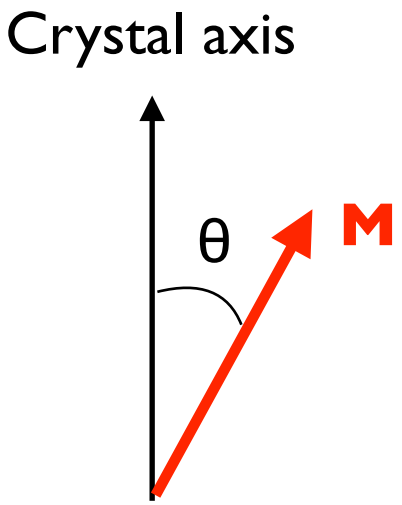
Continuum

$$f_{EX} = \text{const.} + A(|\nabla m_x|^2 + |\nabla m_y|^2 + |\nabla m_z|^2)$$

$$\mathbf{m} = \frac{\mathbf{M}}{M_s}$$ normalized magnetization

Anisotropy energy

magnetocrystalline

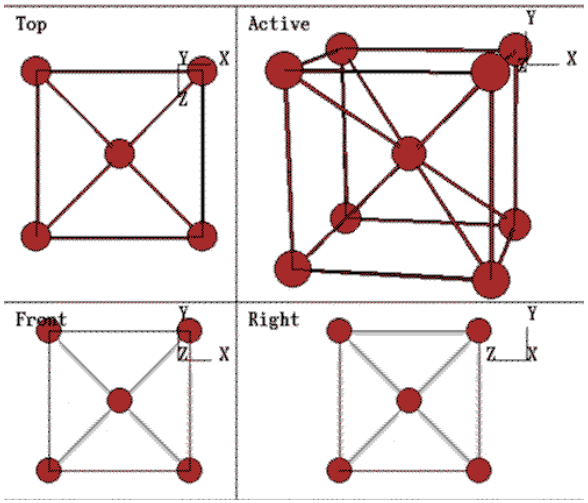


$K_1 \sin^2\theta$

Anisotropy energy

magnetocrystalline

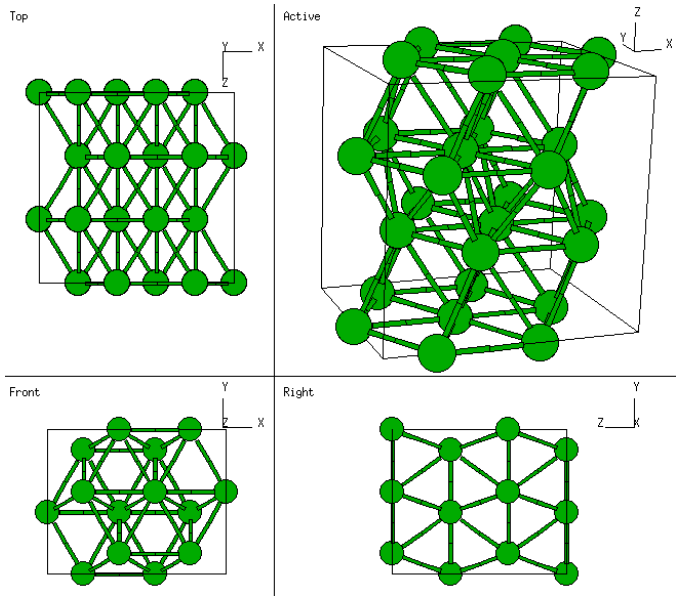
bcc Iron



$K_1 = 0.48 \cdot 10^5$ Jm^{-3}

cubic

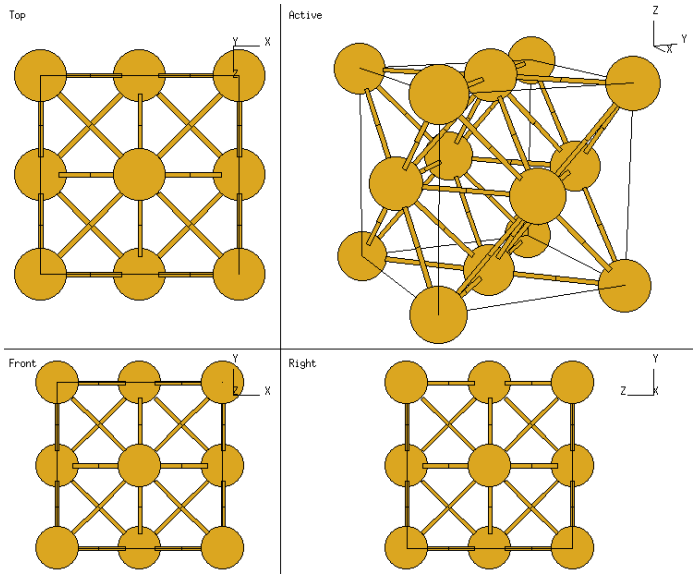
hcp Cobalt



$K_1 = 5 \cdot 10^5$ Jm^{-3}

uniaxial

fcc Nickel



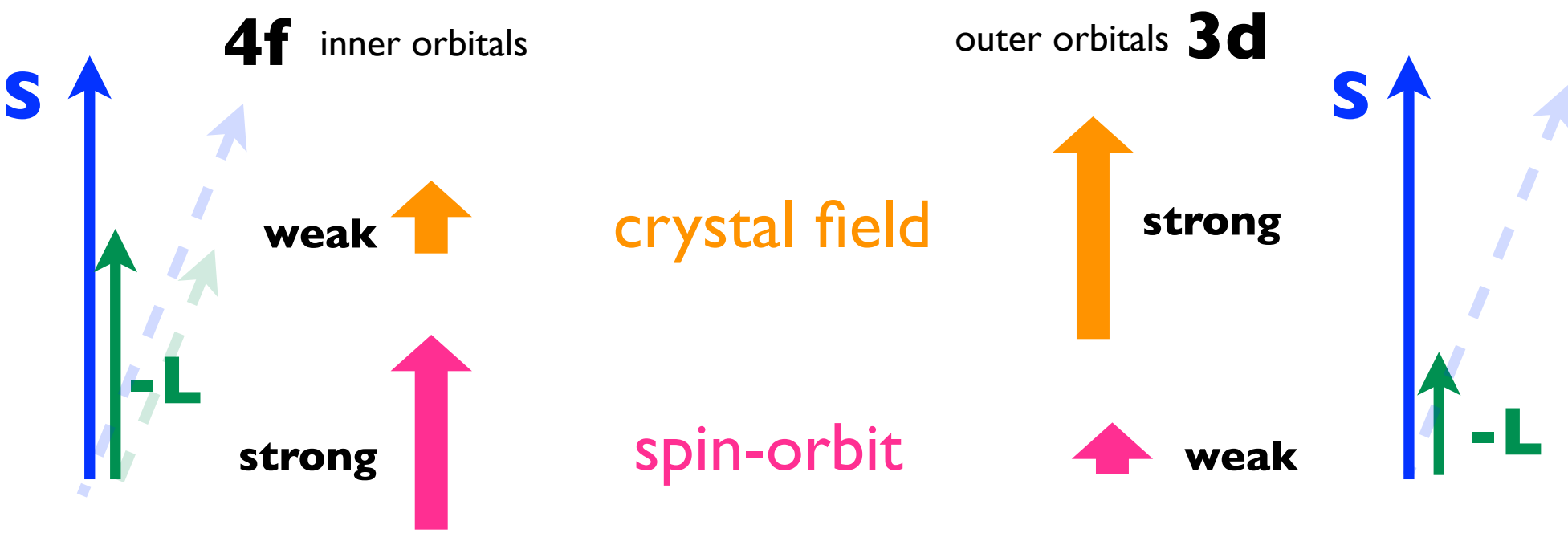
$K_1 = -0.057 \cdot 10^5$ Jm^{-3}

cubic

Anisotropy energy

L is linked to the crystal axis crystal field energy

S is linked to **L** spin orbit energy **L·S**



Energy terms of a ferromagnt

$$G_L = \int_V \left(A(\nabla \mathbf{m})^2 + f_{AN}(\mathbf{m}, \mathbf{n}) - \frac{1}{2}\mu_0 \mathbf{H}_M \cdot \mathbf{M} - \mu_0 \mathbf{H}_a \cdot \mathbf{M} \right) d^3 r$$

exchange

anisotropy

magnetostatic field

applied field

thanks for your attention !

Bibliography

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