# Magnetization processes

*Vittorio Basso, INRIM, v.basso@inrim.it*

# Outline

**The static hysteresis loop**

- Domain wall motion

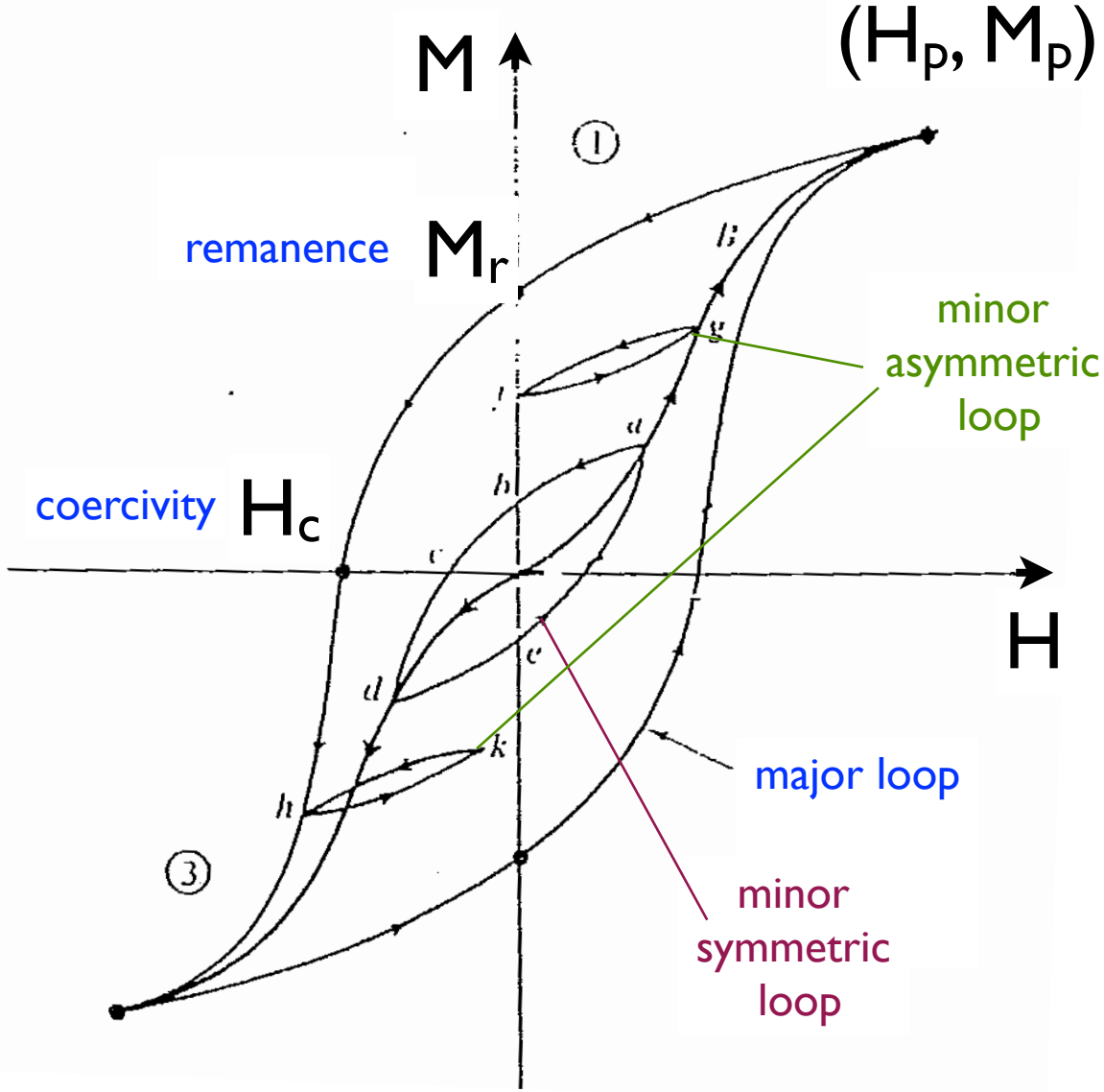
**The dynamic hysteresis loop**

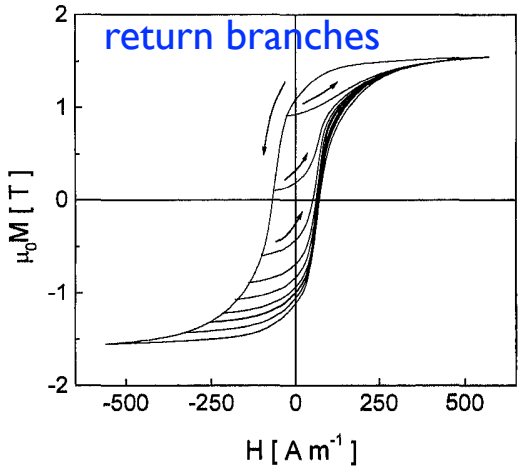
- Eddy currents and the theory of losses

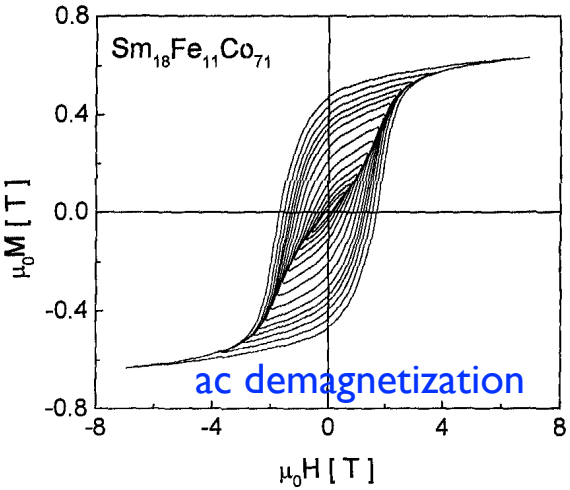
# Hysteresis loop

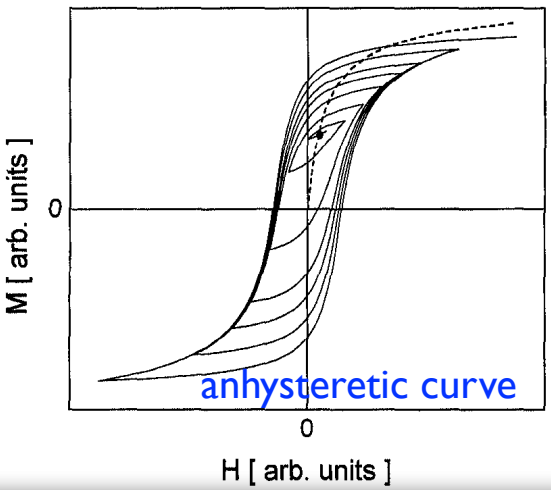
rate independent hysteresis

static processes



# Hysteresis loop

rate independent hysteresis

Free energy

$$\frac{\partial f}{\partial t} = -s\frac{\partial T}{\partial t} + \mu_0 \mathbf{H} \cdot \frac{\partial \mathbf{M}}{\partial t} - T\sigma_s$$

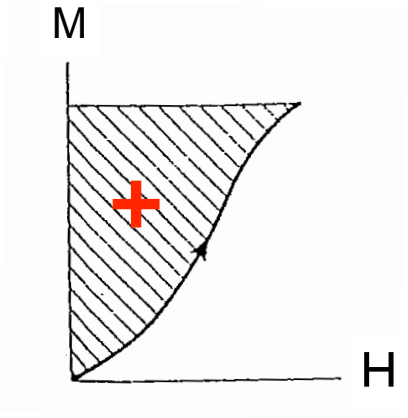
dissipated energy

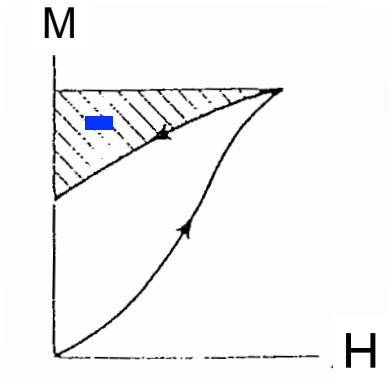
for isothermal processes

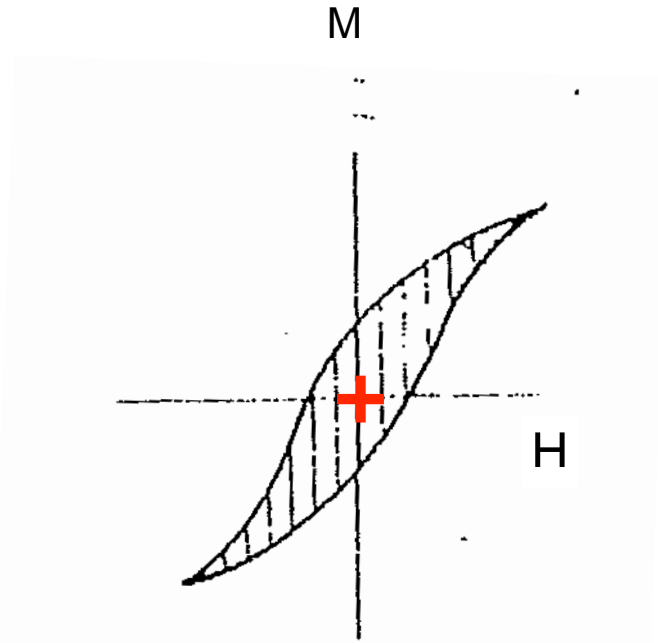
$$df = \mu_0 \mathbf{H} \cdot d\mathbf{M} - T\sigma_s dt$$

work dissipated energy

along the hysteresis loop



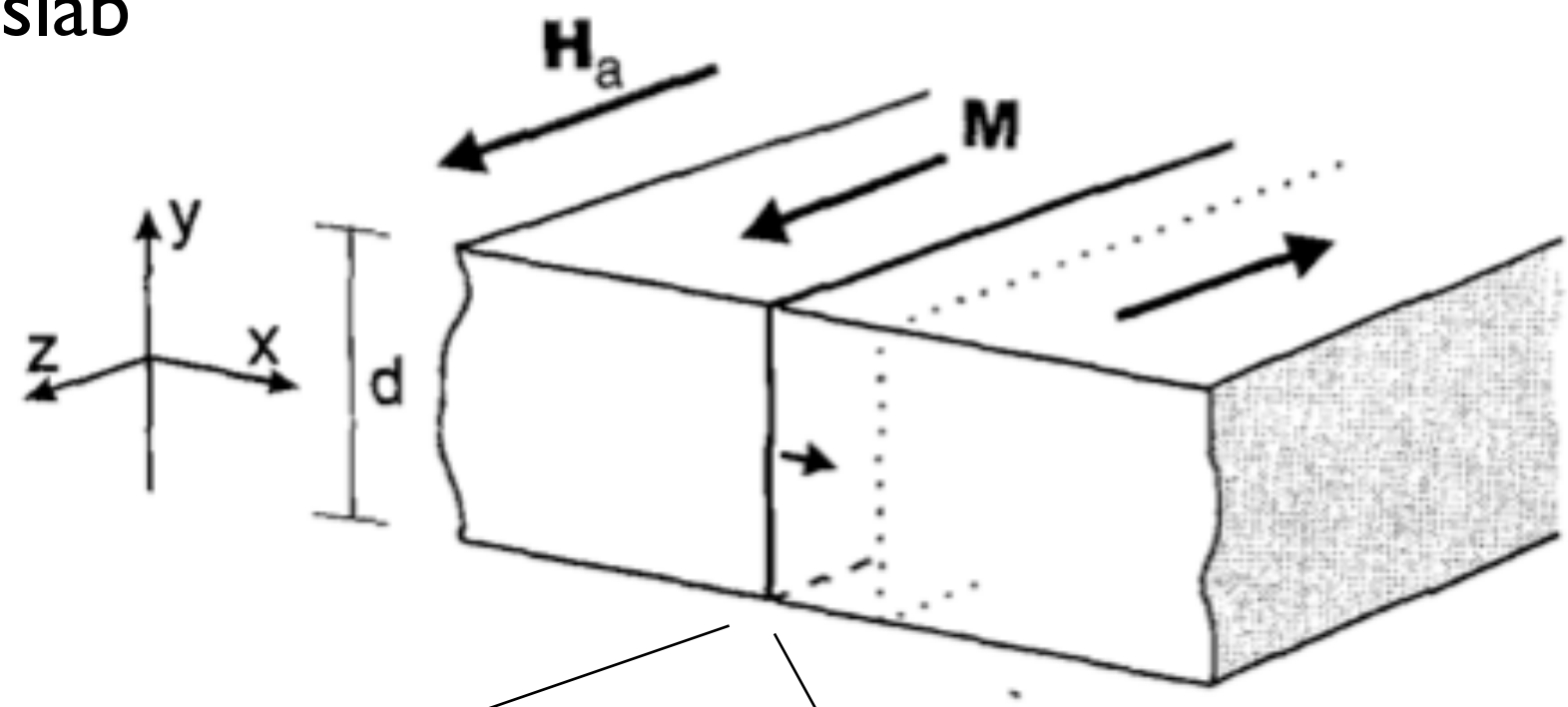



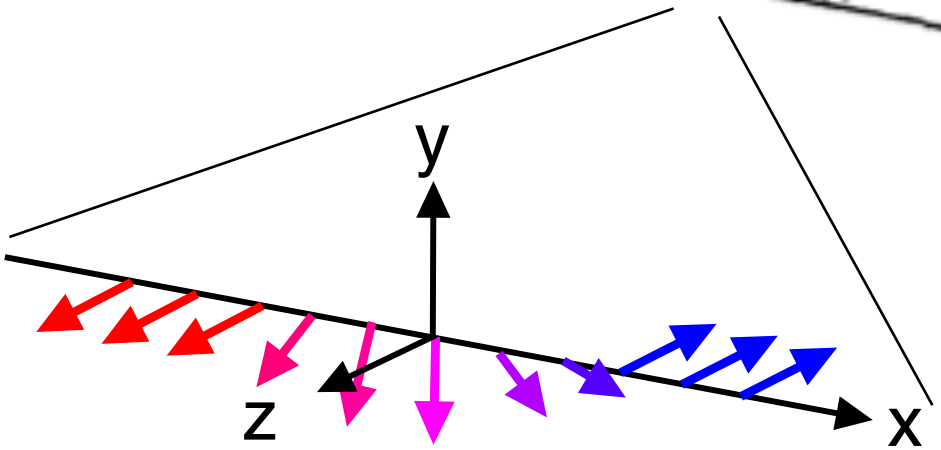

energy losses

**Why do we have energy dissipation with static processes?**

# Domain wall motion

slab




180° Bloch domain wall

$$l_W = \sqrt{\frac{A}{K}}$$

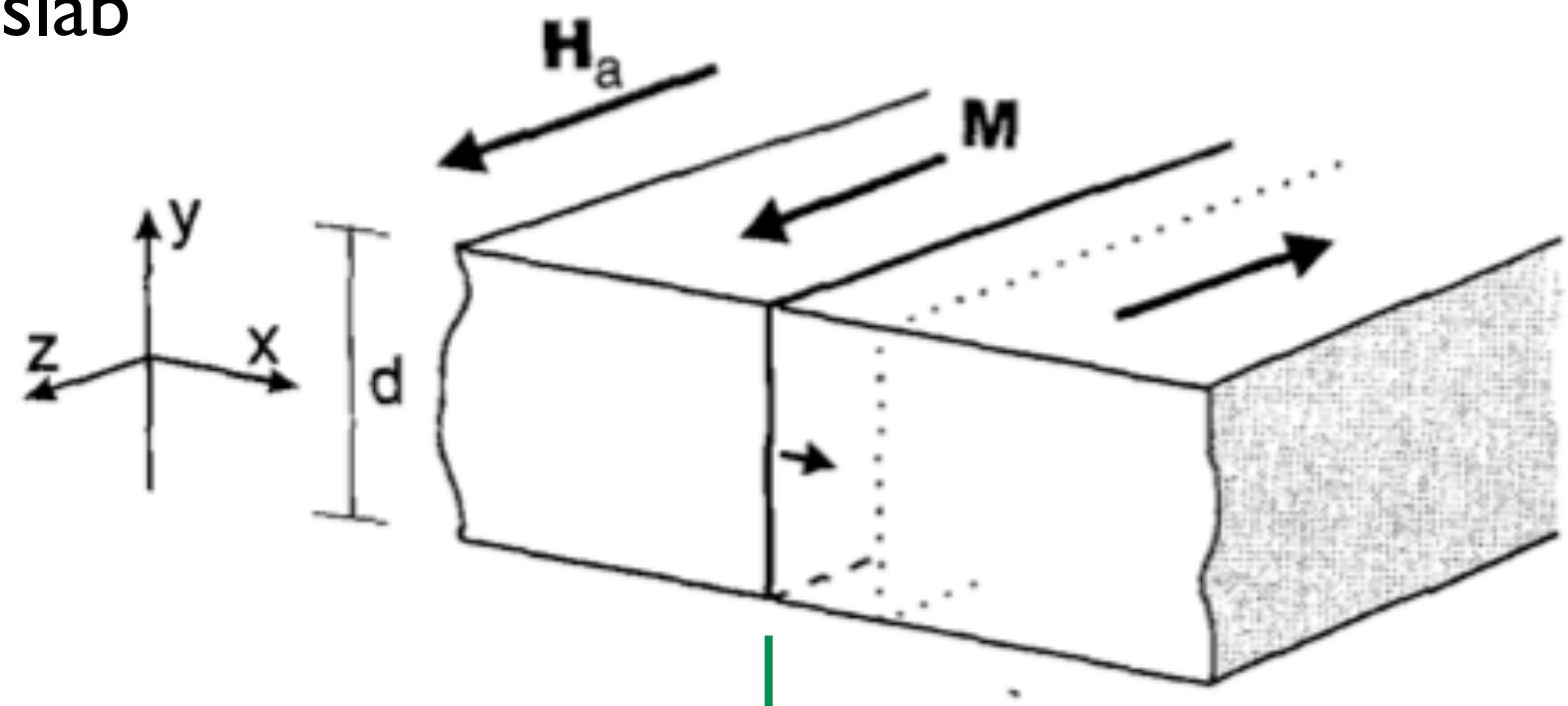
exchange stiffness

anisotropy

50 nm

# Domain wall motion

slab

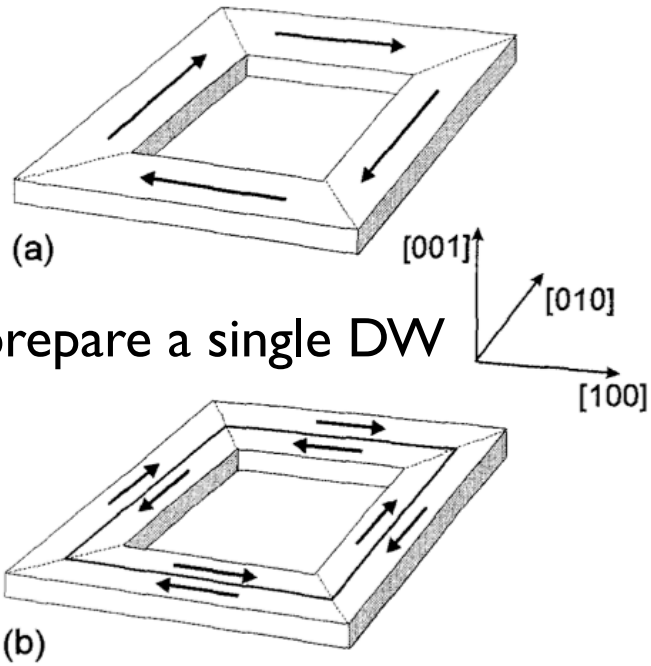



180° Bloch domain wall

position *x* of the DW

# Domain wall motion

1) Fe-Si single crystal picture frame

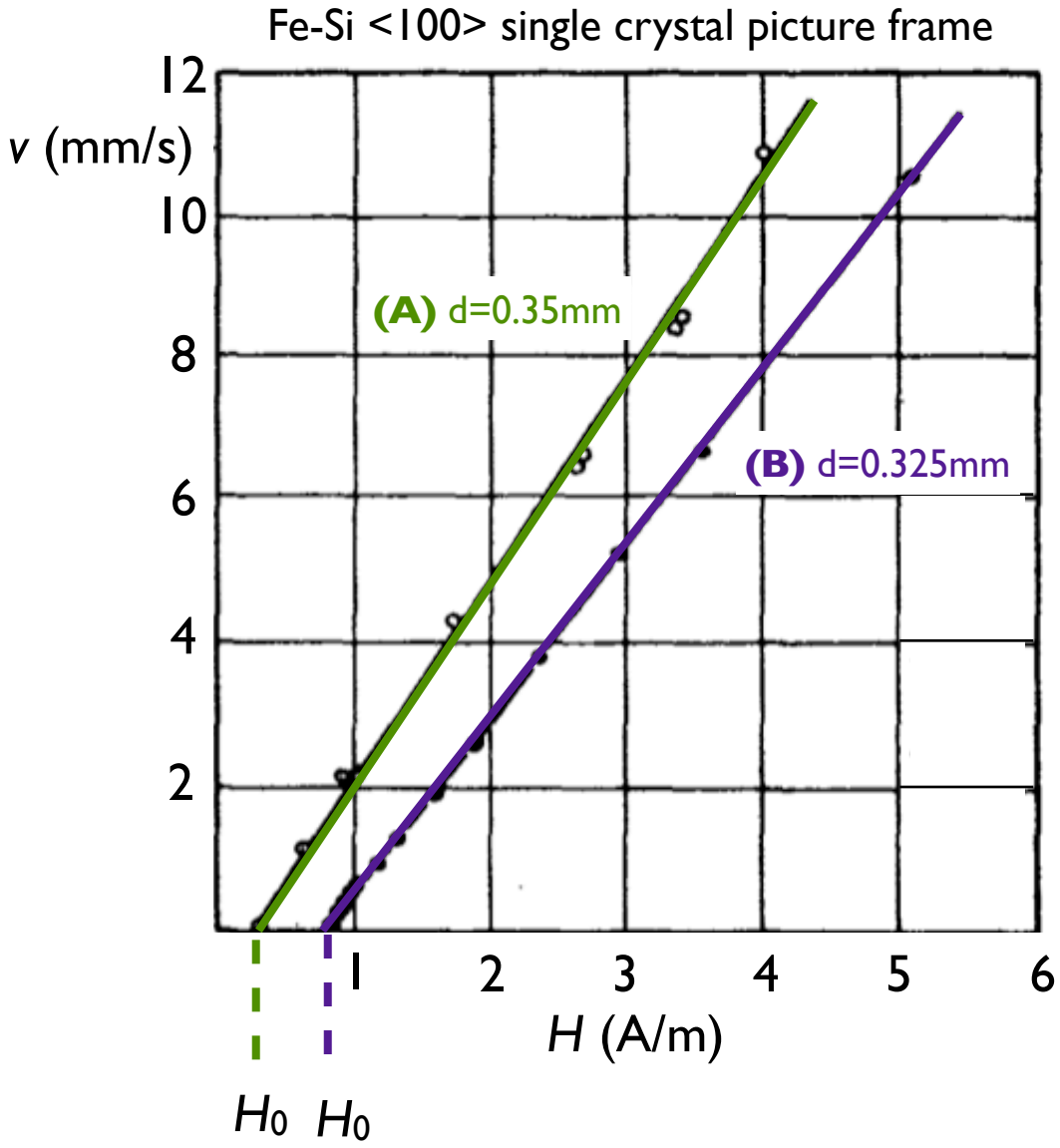



2) prepare a single DW




3) apply the field $H$

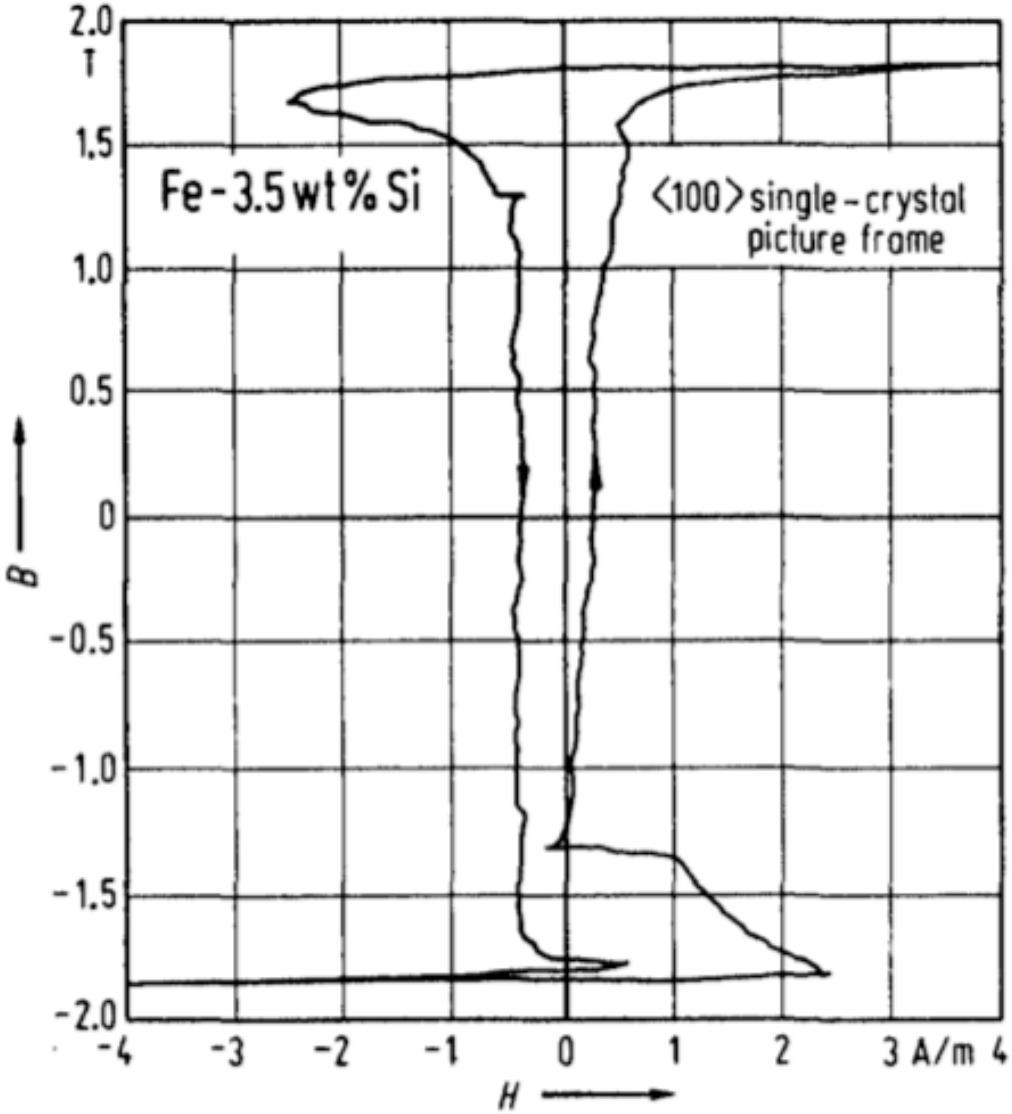
4) velocity proportional to $H$
but with the threshold $H_0$

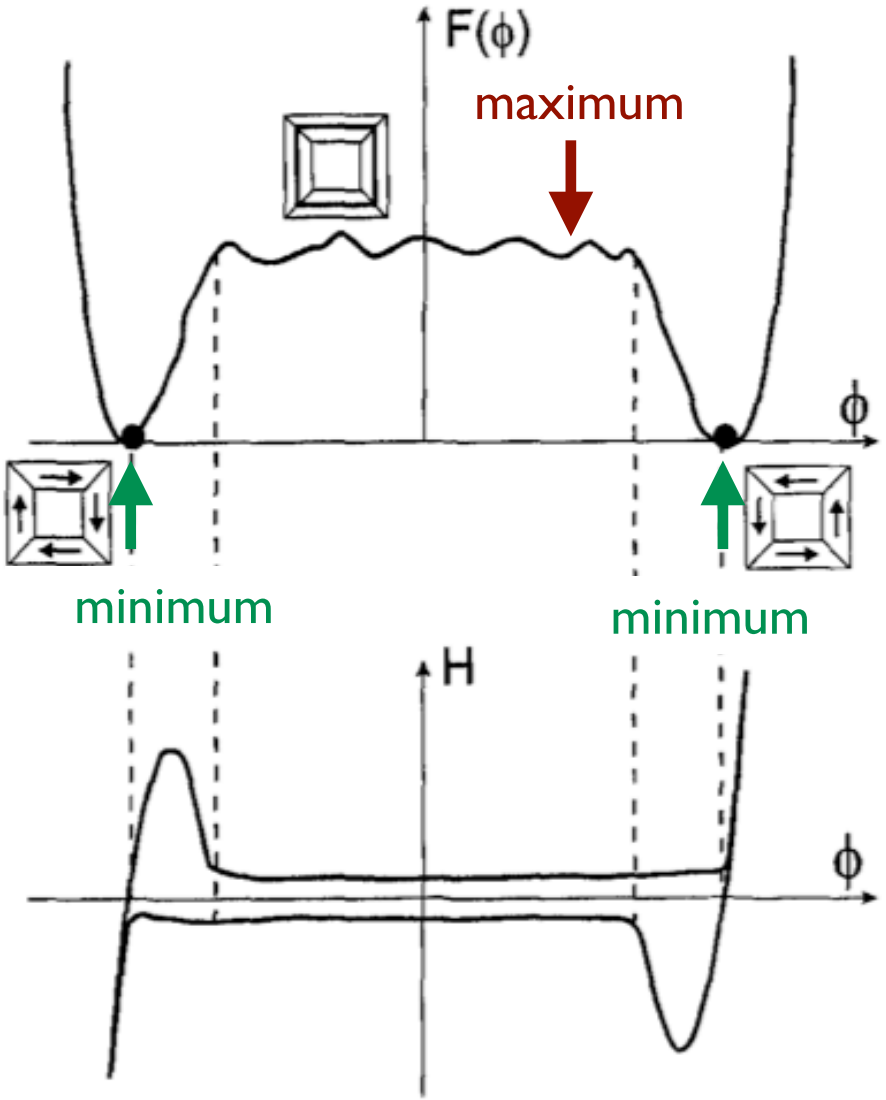



$$v = a\,(H\text{-}H_0)$$

# Domain wall motion

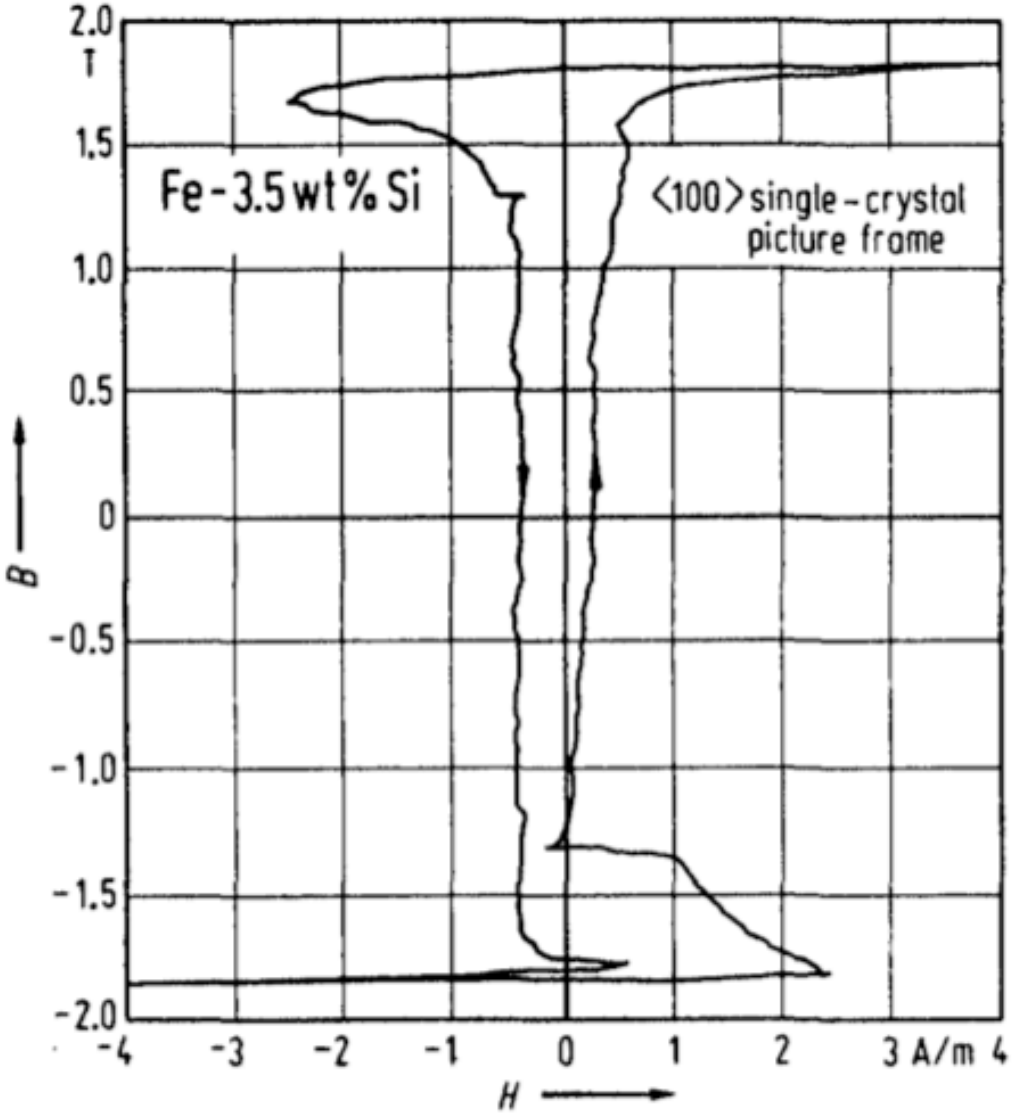
Control *H* to have a constant d*x*/d*t*

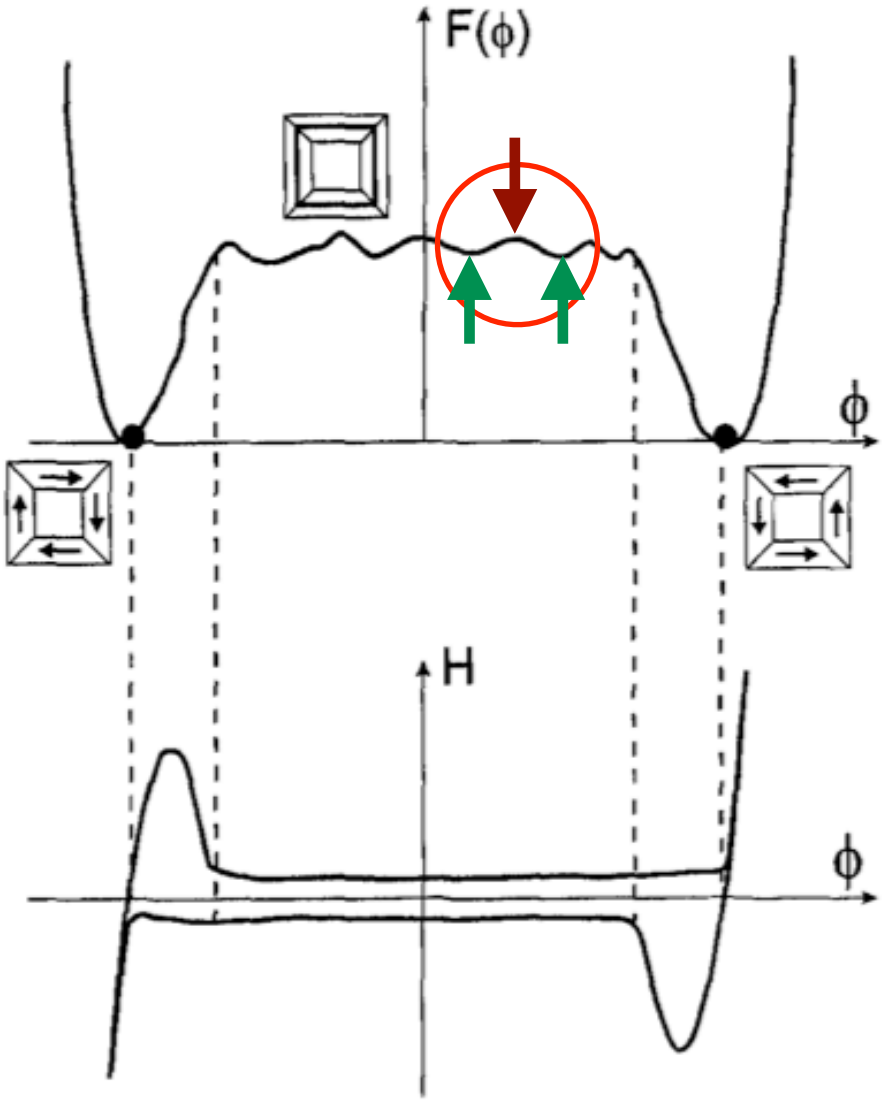


# Domain wall motion

of picture frame



# Hysteresis loop

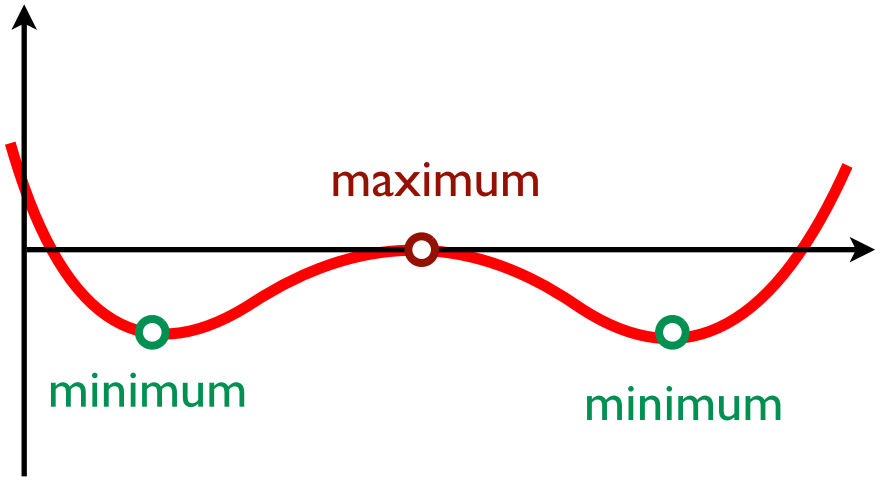
## metastability

$G_L(x)=F_L(x)-Hx$

energy minimum

$$\frac{\partial G_L}{\partial x} = 0$$

$$\frac{\partial F_L}{\partial x} - H = 0$$



# Hysteresis loop

## metastability

$G_L(x)=F_L(x)-Hx$

energy minimum

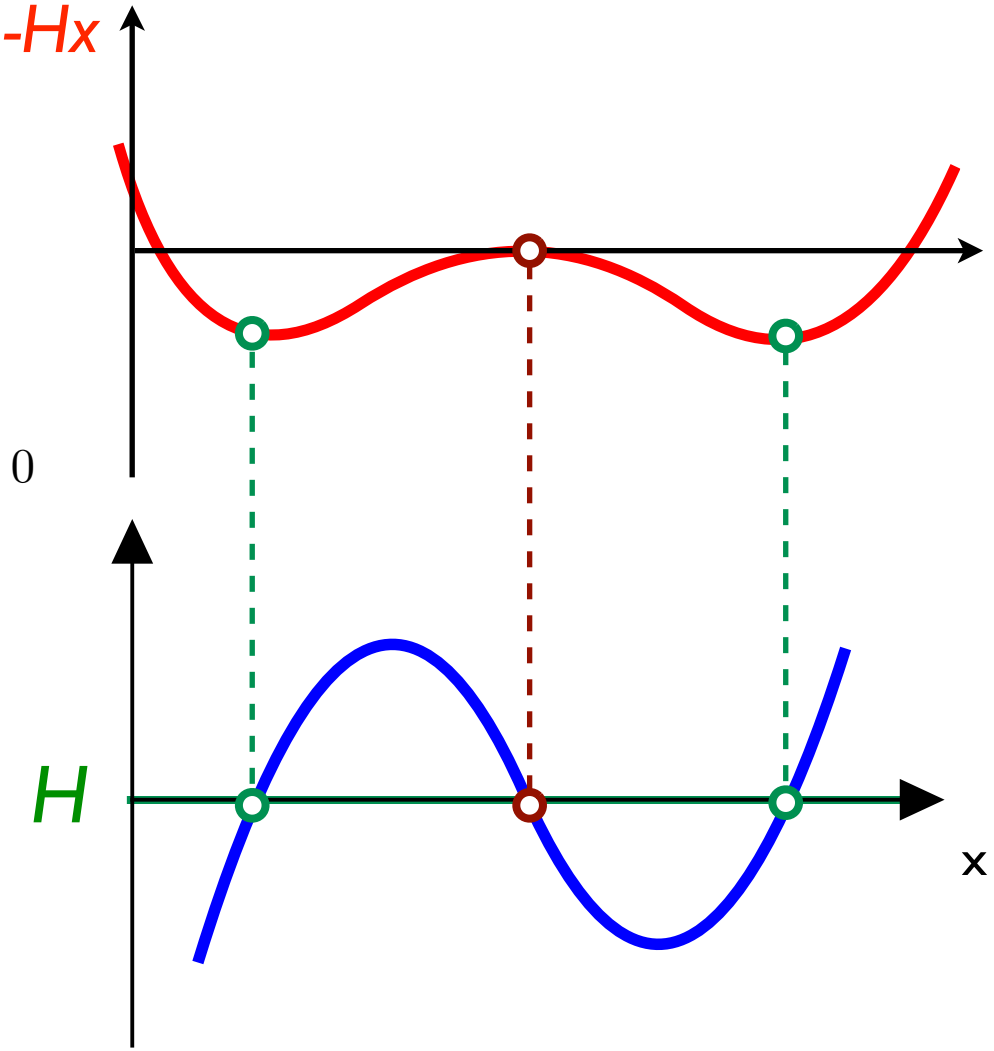
$$\frac{\partial G_L}{\partial x} = 0$$

$$\frac{\partial F_L}{\partial x} - H = 0$$

pinning field

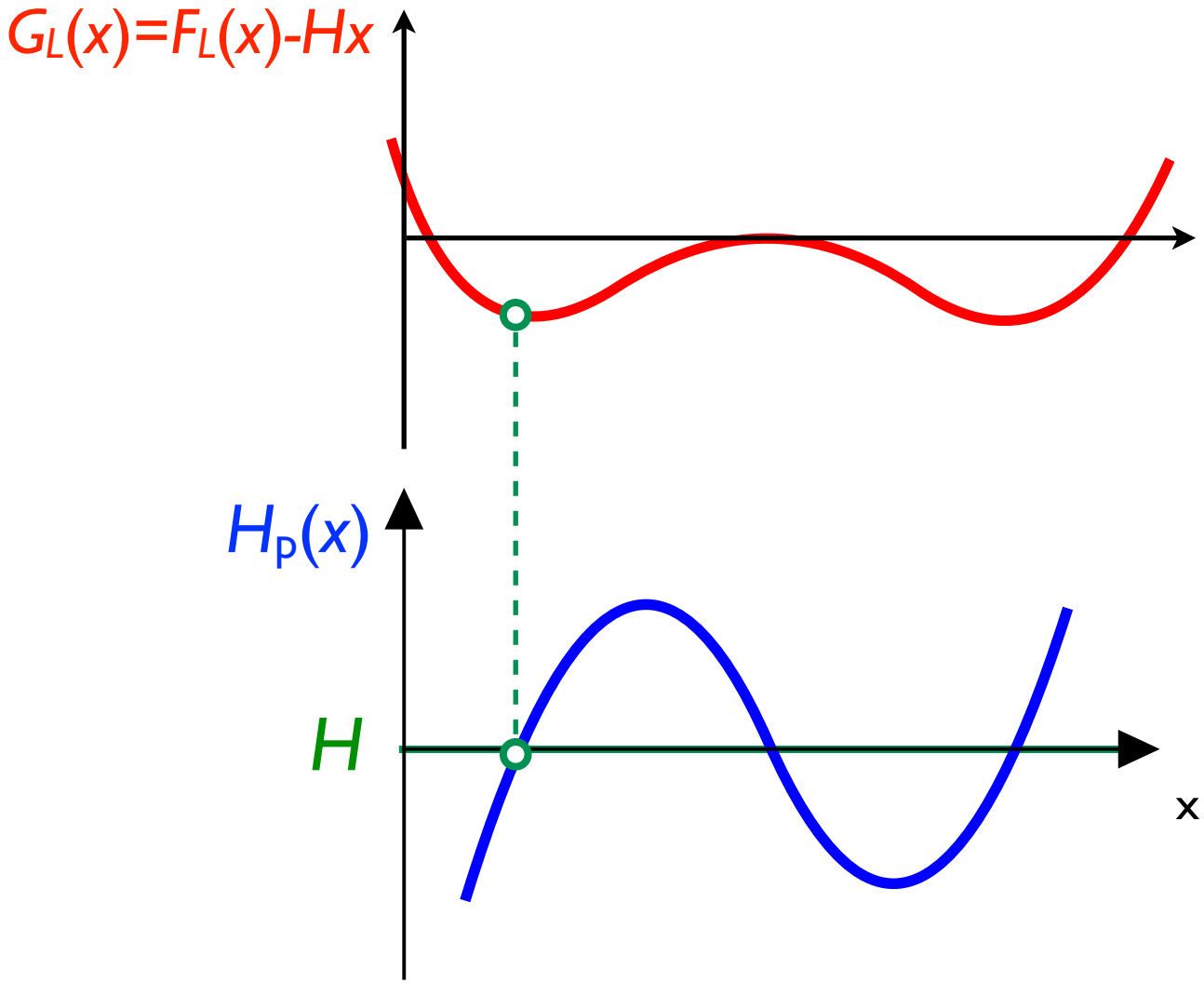
$$H_p(x) = \frac{\partial F_L}{\partial x}$$

$$H = H_p(x)$$



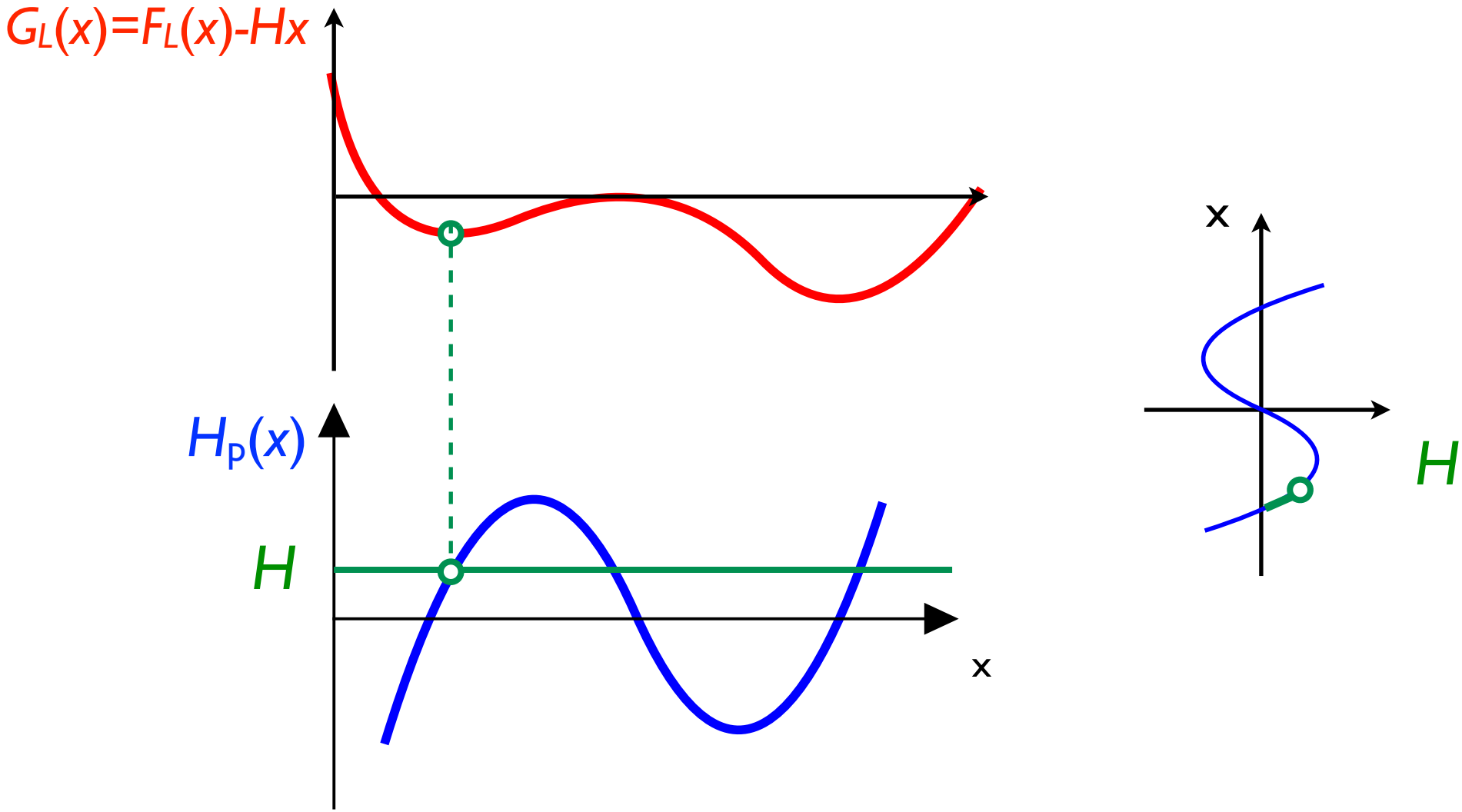
# Hysteresis loop

## metastability



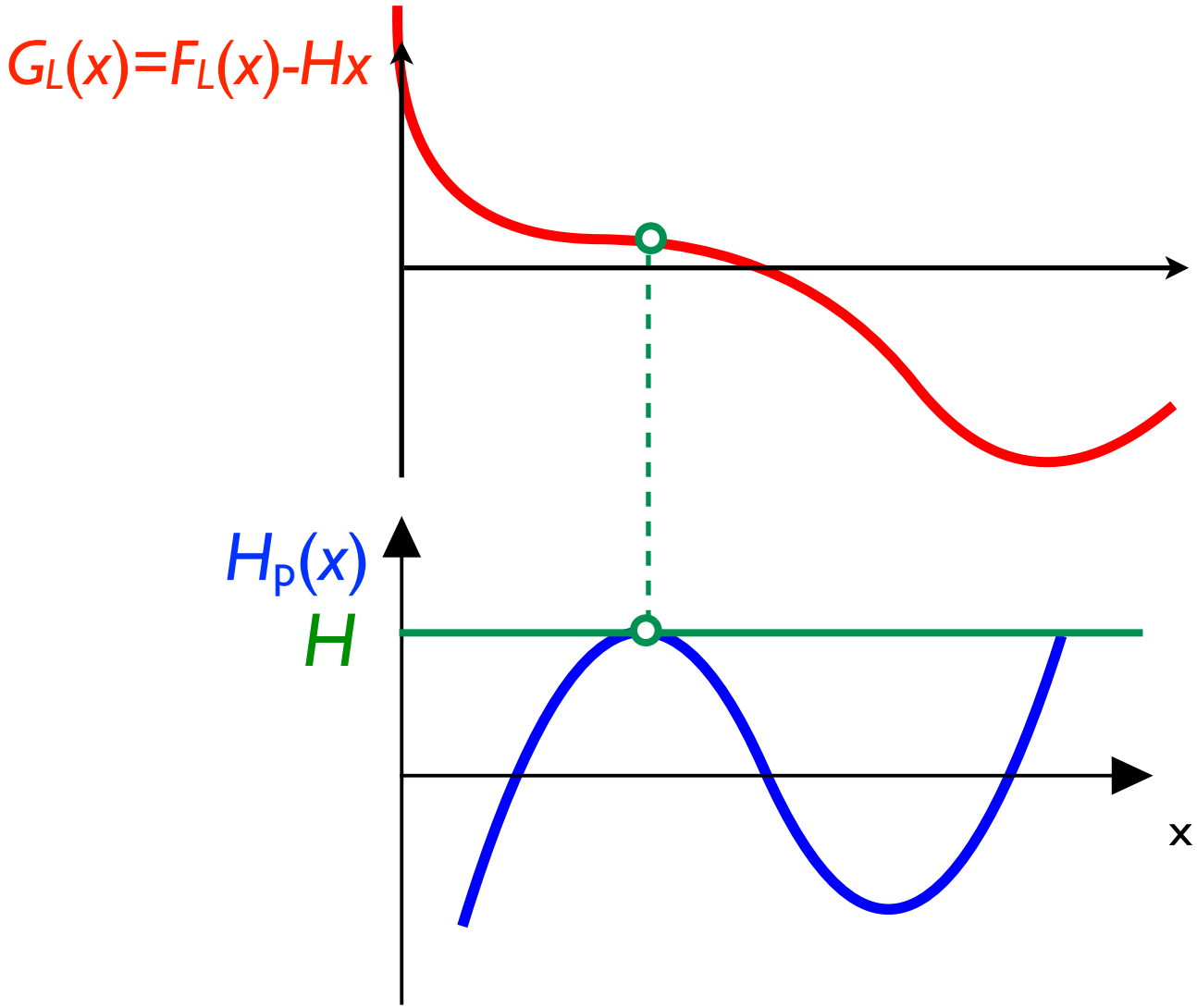
# Hysteresis loop

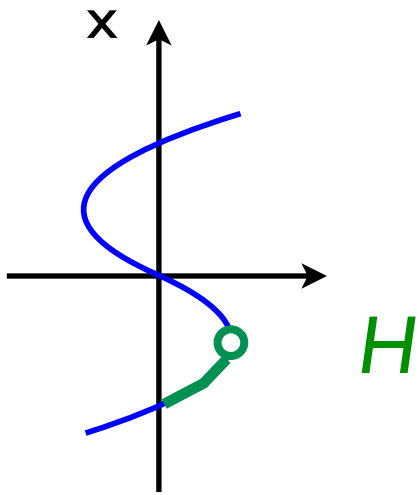
## metastability



# Hysteresis loop

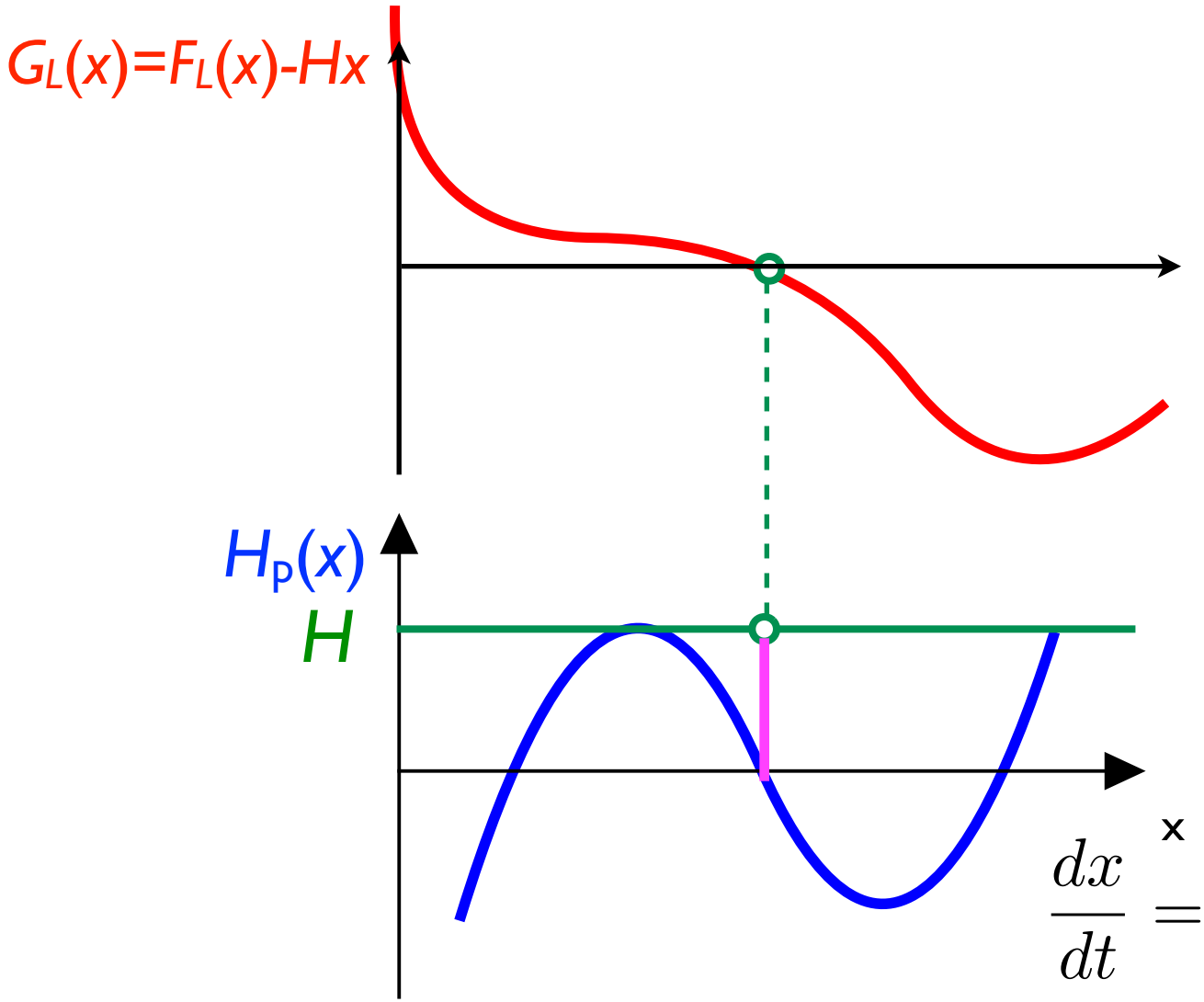
## metastability

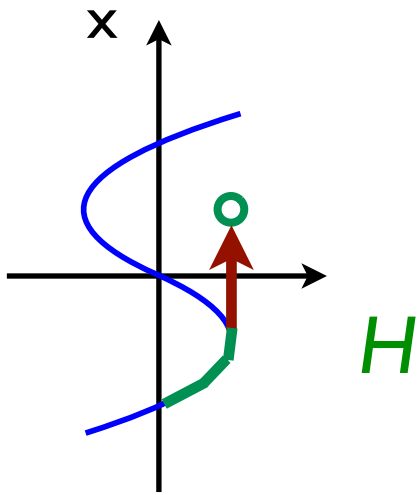


# Hysteresis loop

## metastability



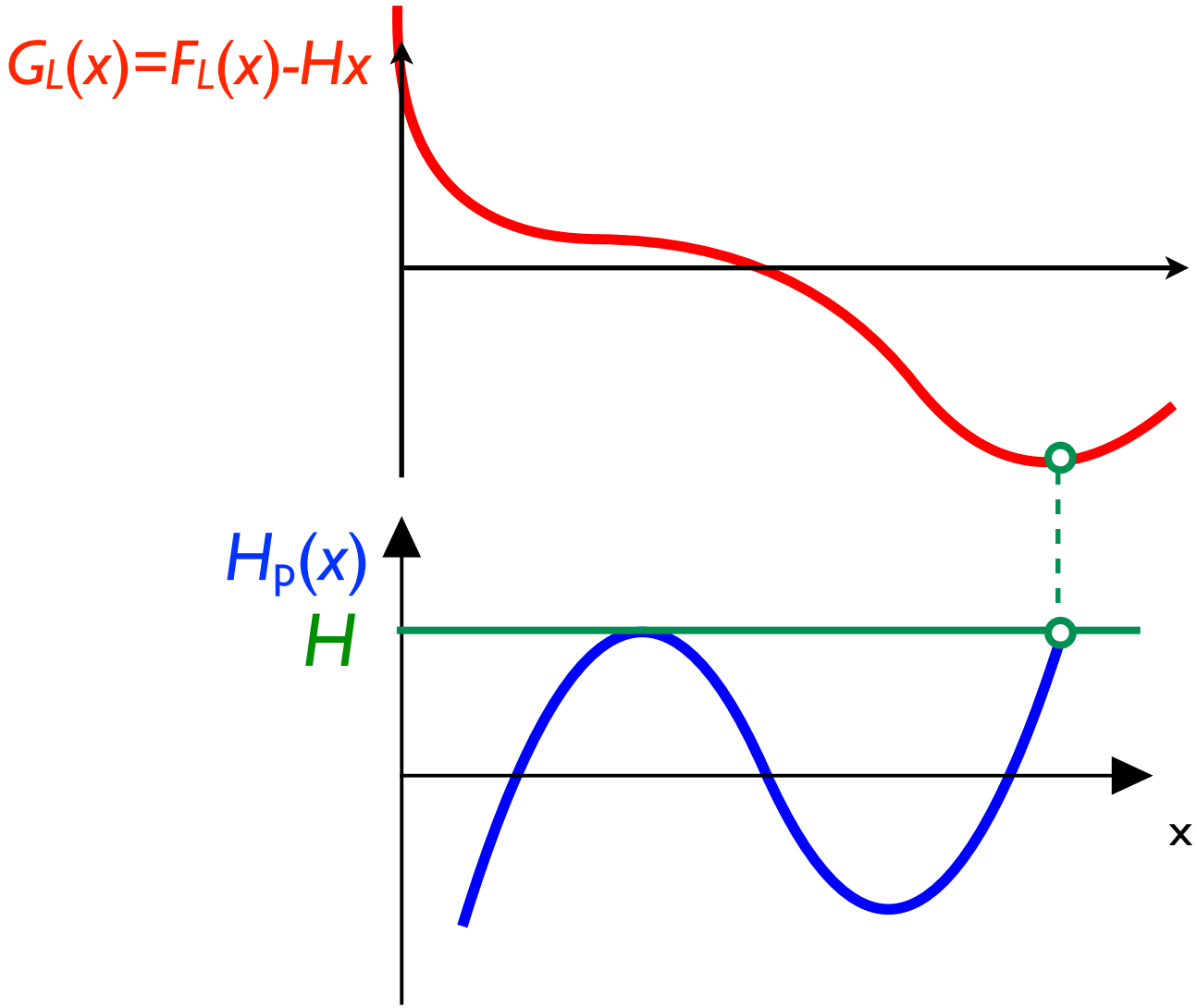



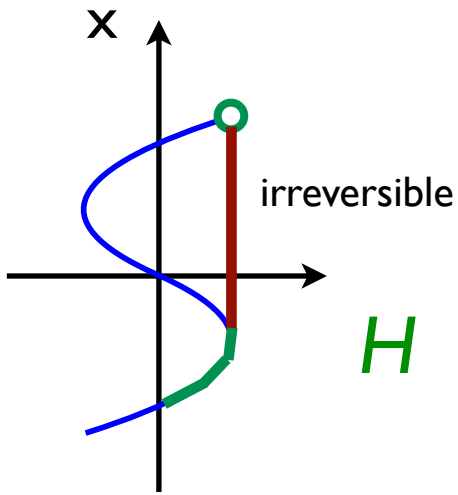

$$\frac{dx}{dt} = \alpha[H - H_p(x)]$$

velocity force

# Hysteresis loop

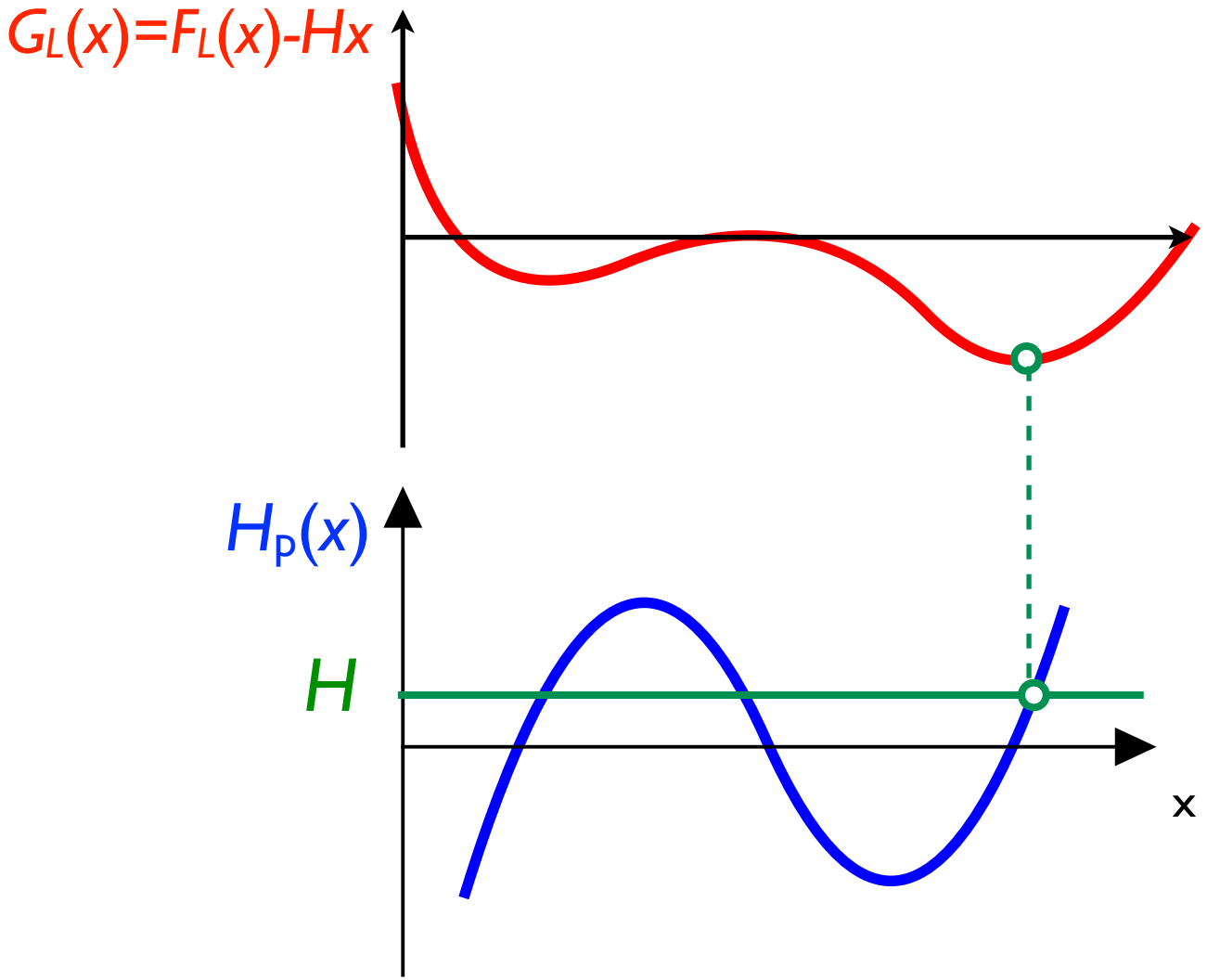
## metastability

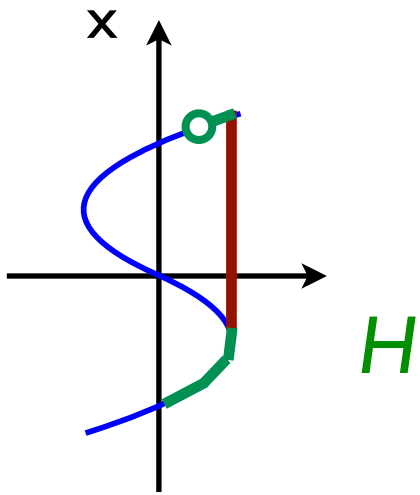


# Hysteresis loop

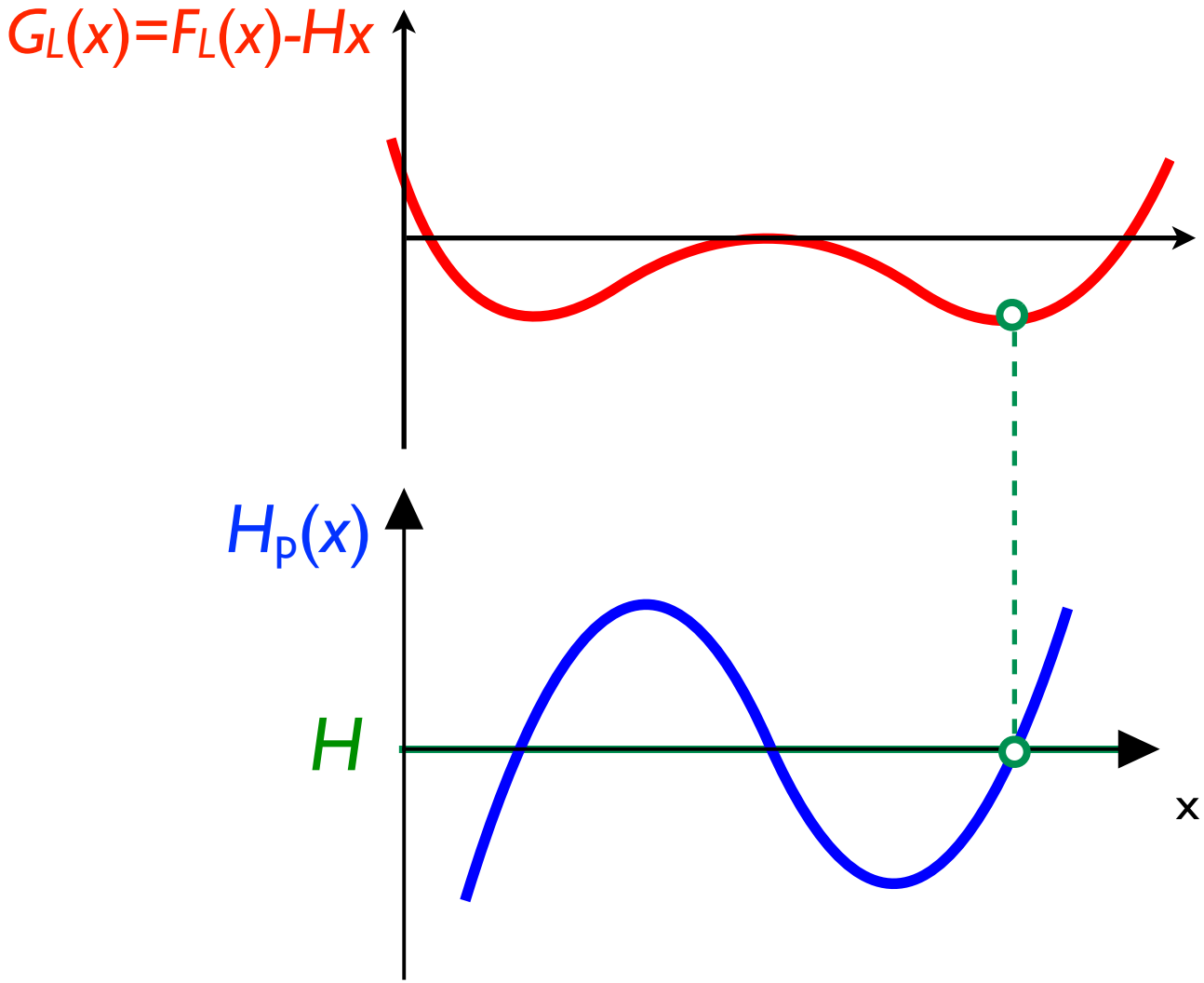
## metastability

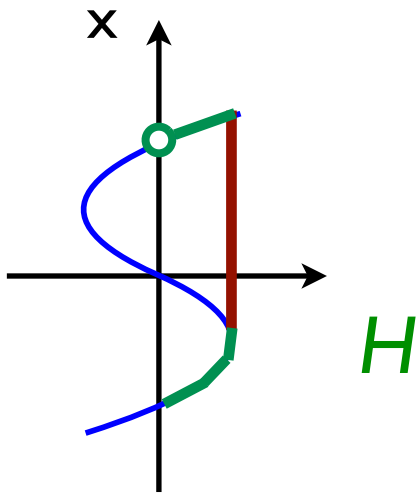


# Hysteresis loop

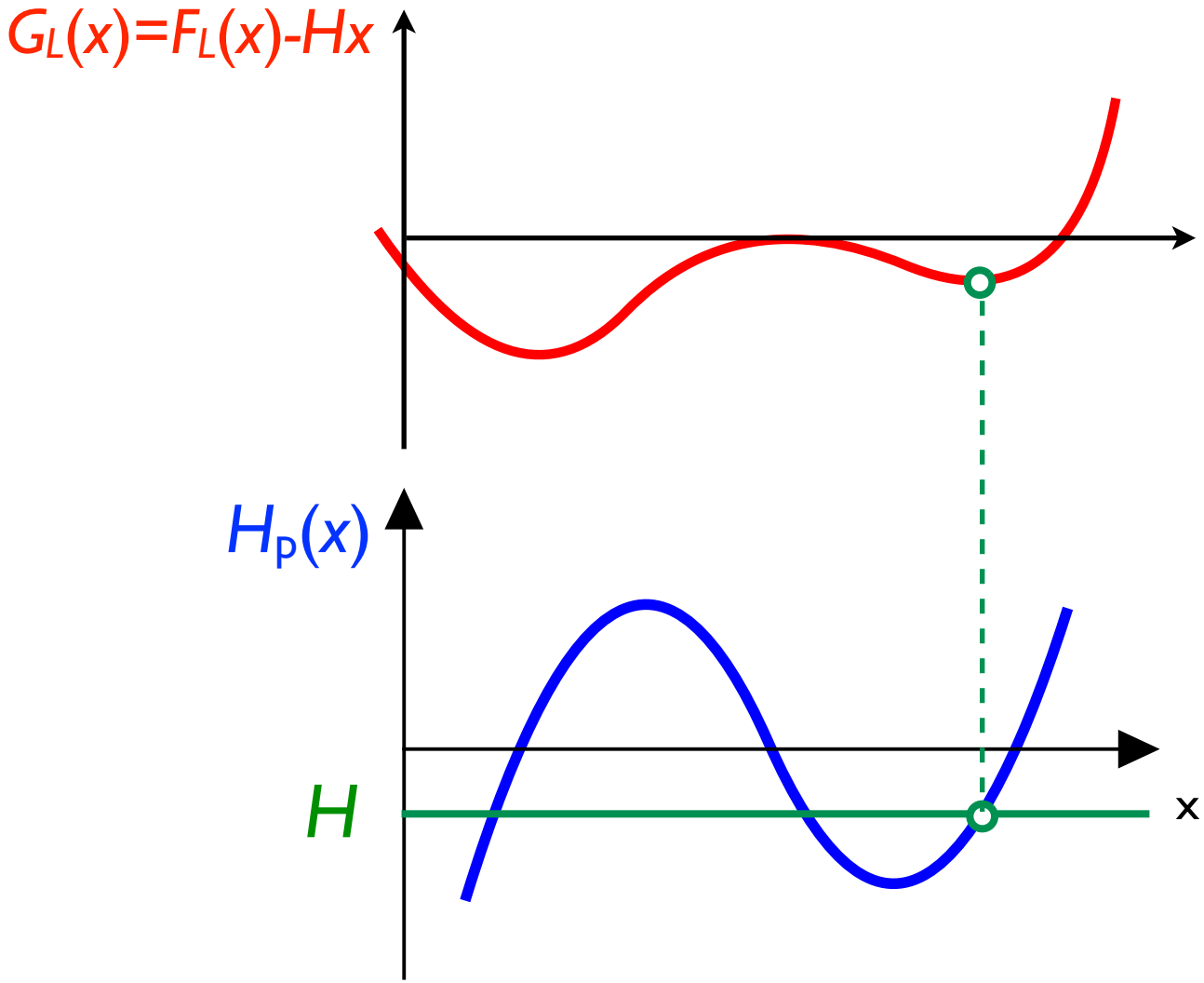
## metastability

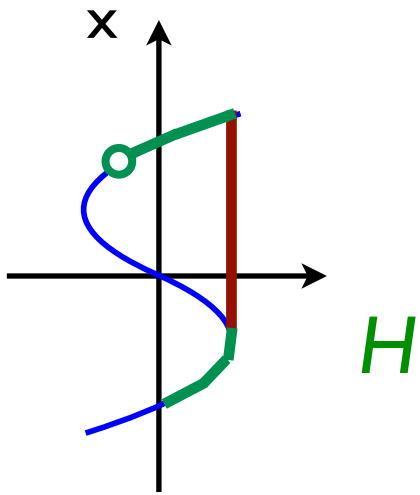


# Hysteresis loop

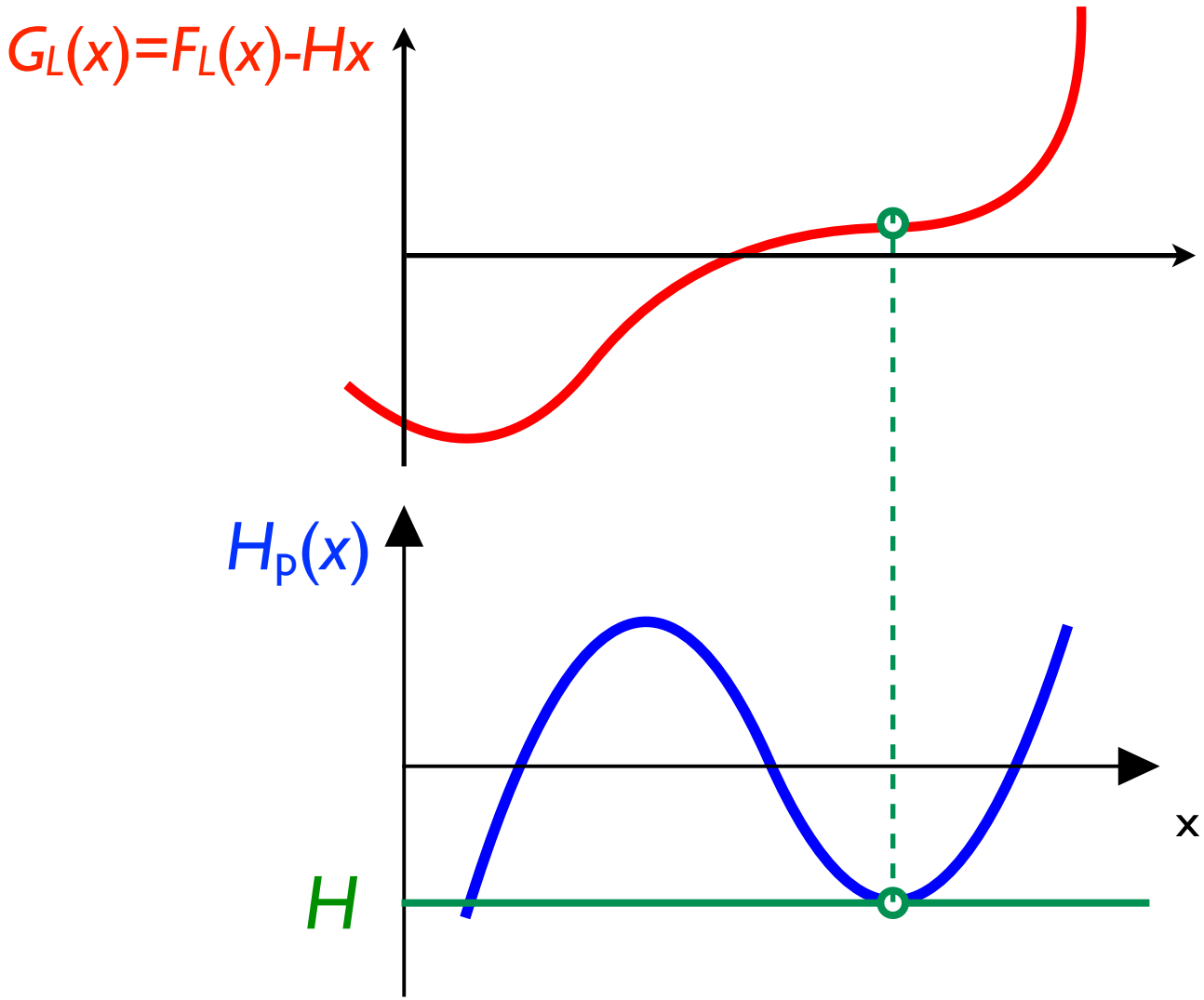
## metastability

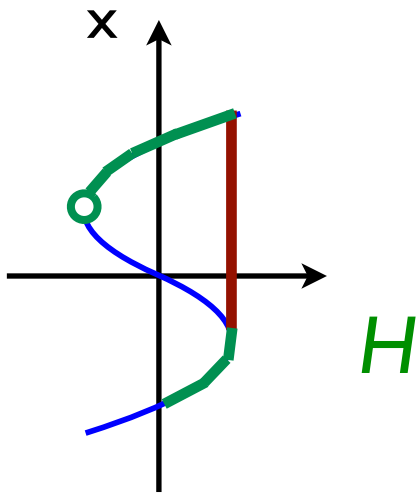


# Hysteresis loop

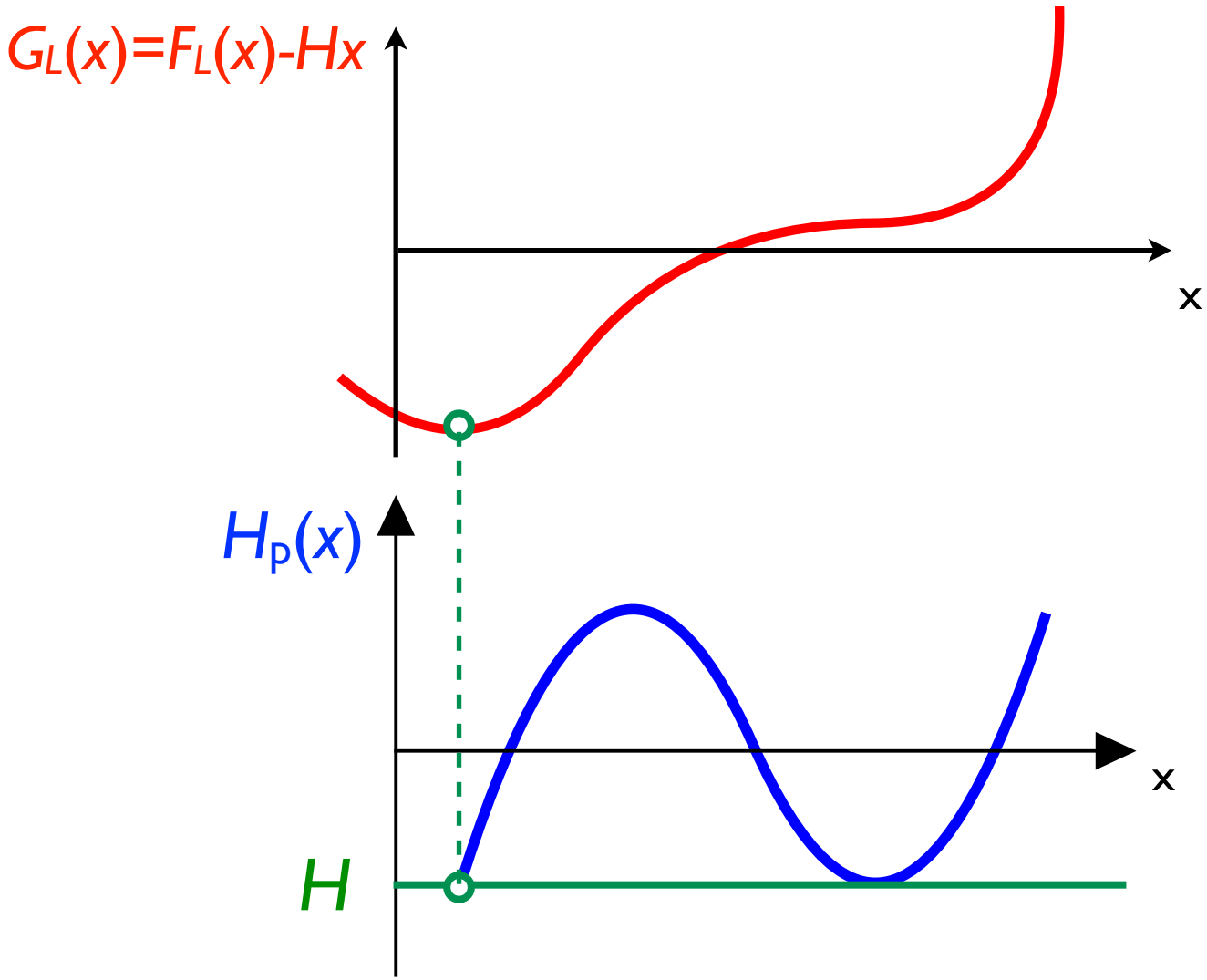
## metastability

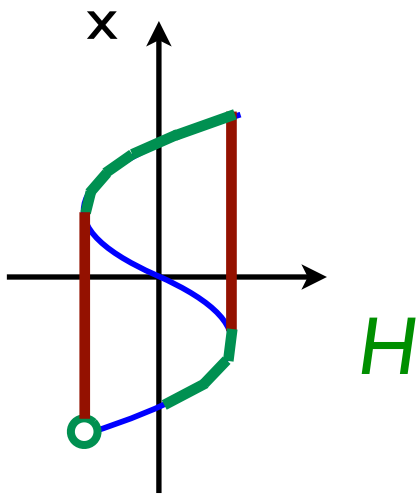


# Hysteresis loop

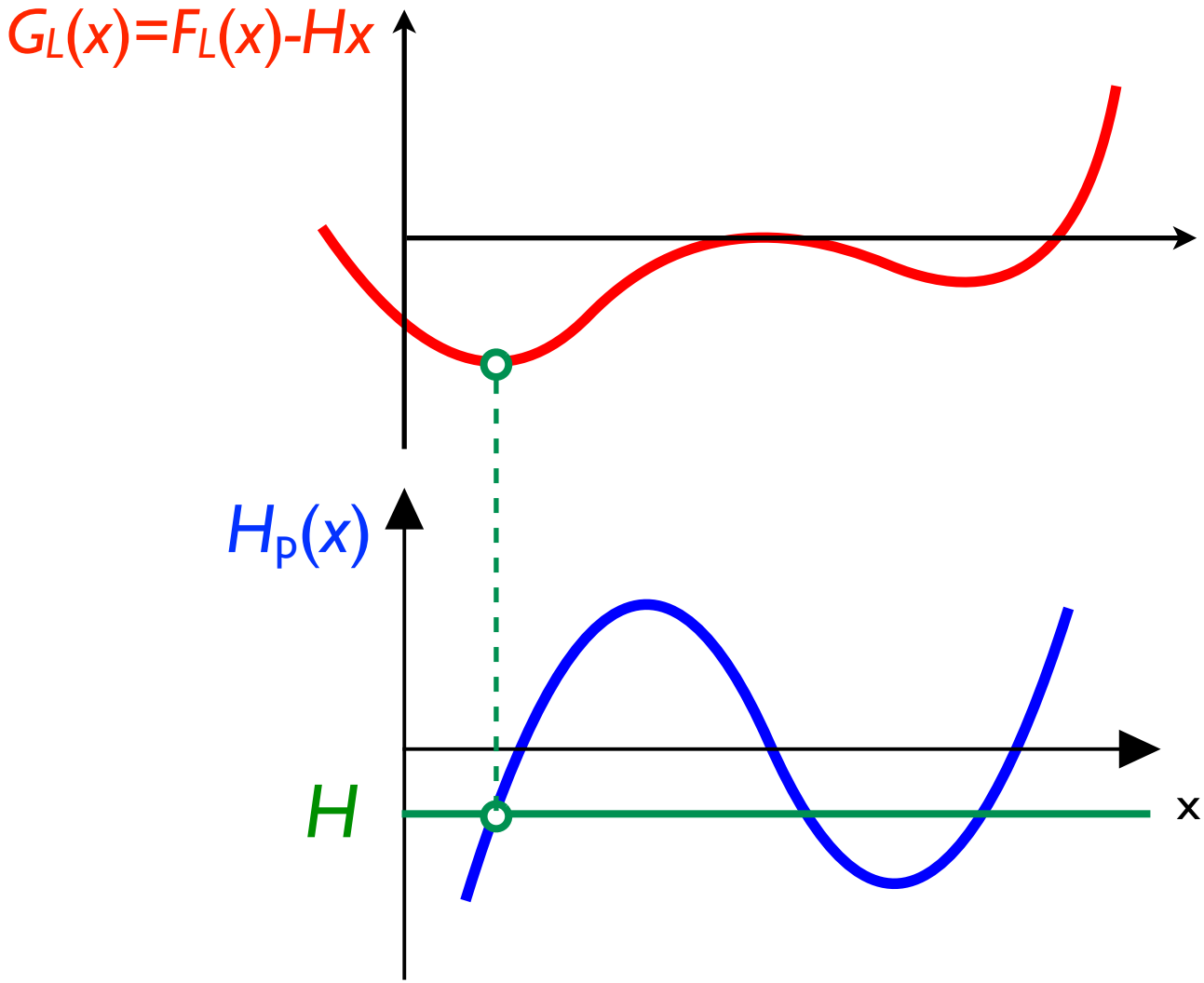
## metastability

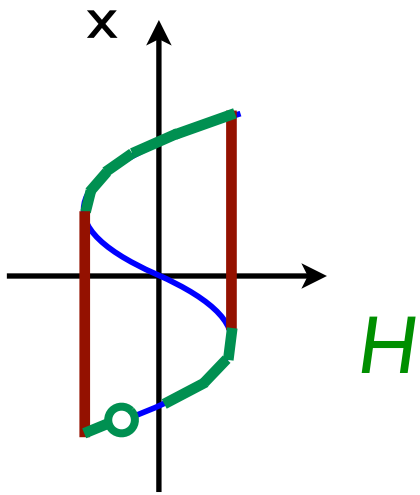


# Hysteresis loop

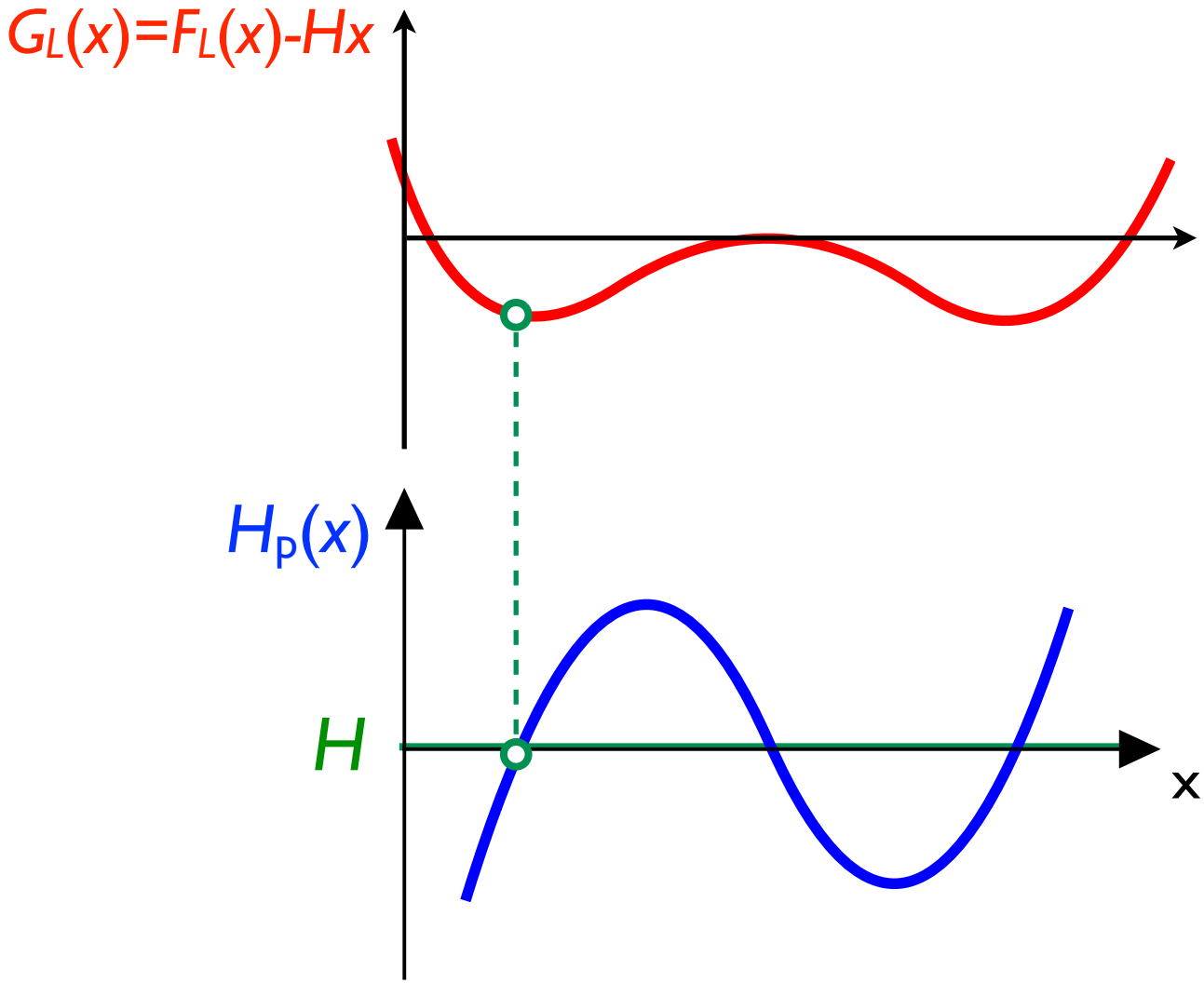
## metastability

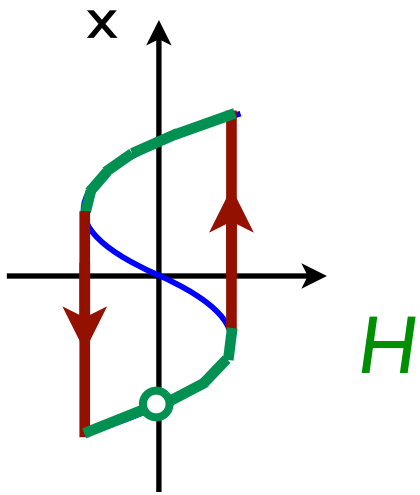


# Hysteresis loop

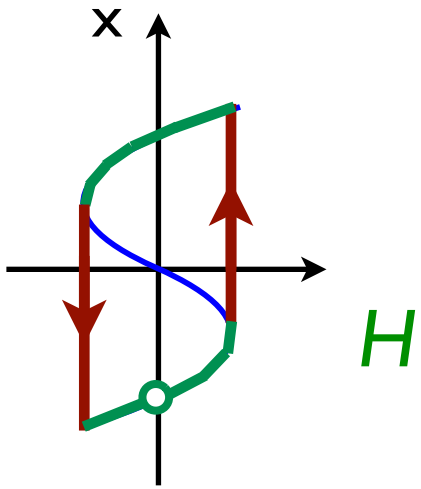
## metastability



# Hysteresis loop

## dissipation and metastability



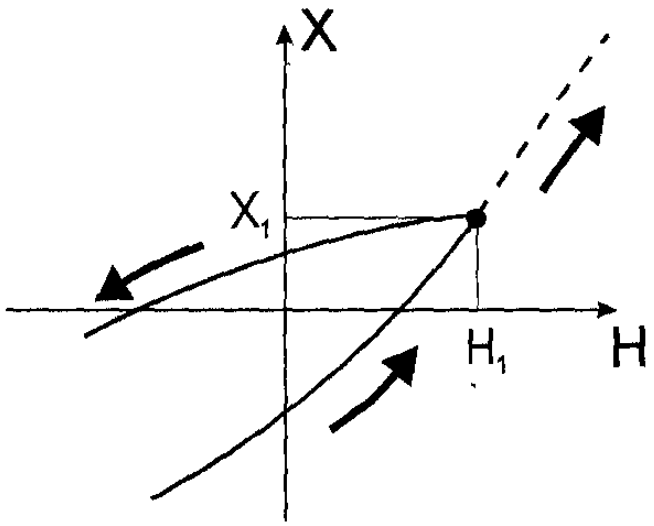

metastable states

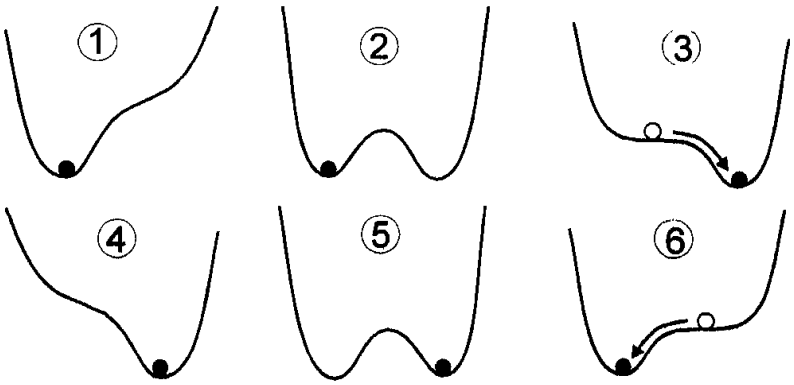
$$\frac{dx}{dt} = \alpha[H - H_p(x)]$$

dissipation mechanism

# Hysteresis loop

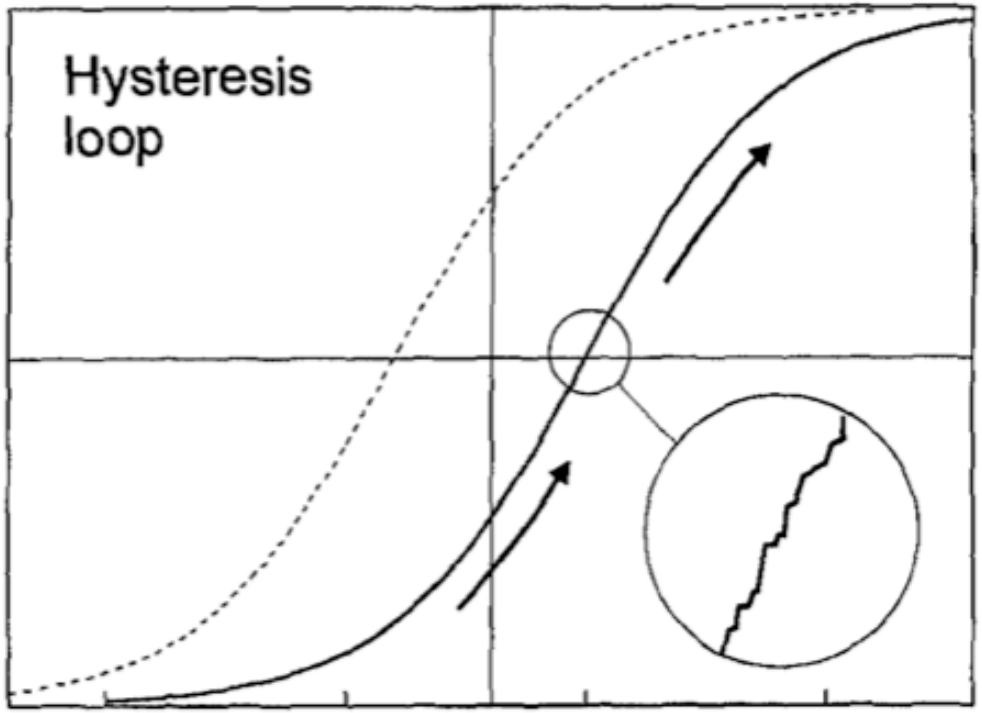
## branching and metastability

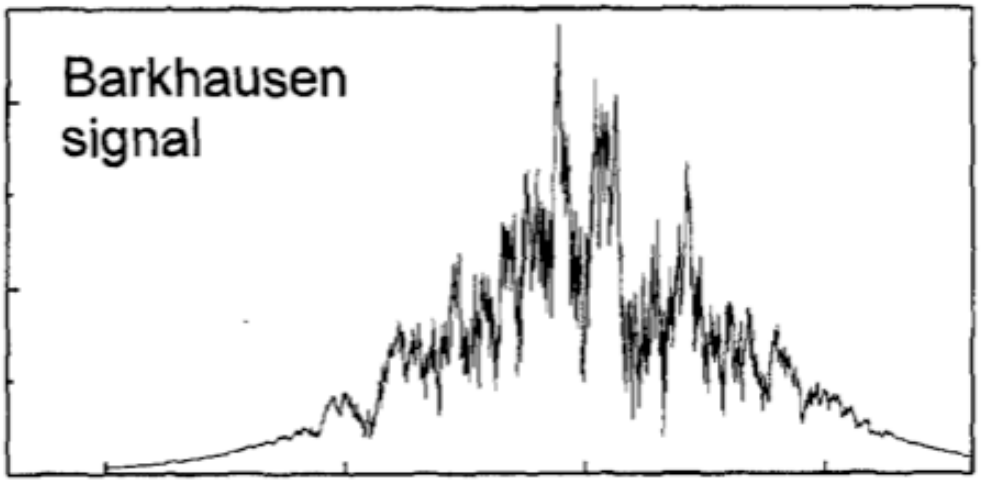


**How to form continuous hysteresis branches?**

# Hysteresis loop




**static hysteresis branches are never continuous!**

# Hysteresis loop

**static hysteresis**

hysteresis loop

**metastability**

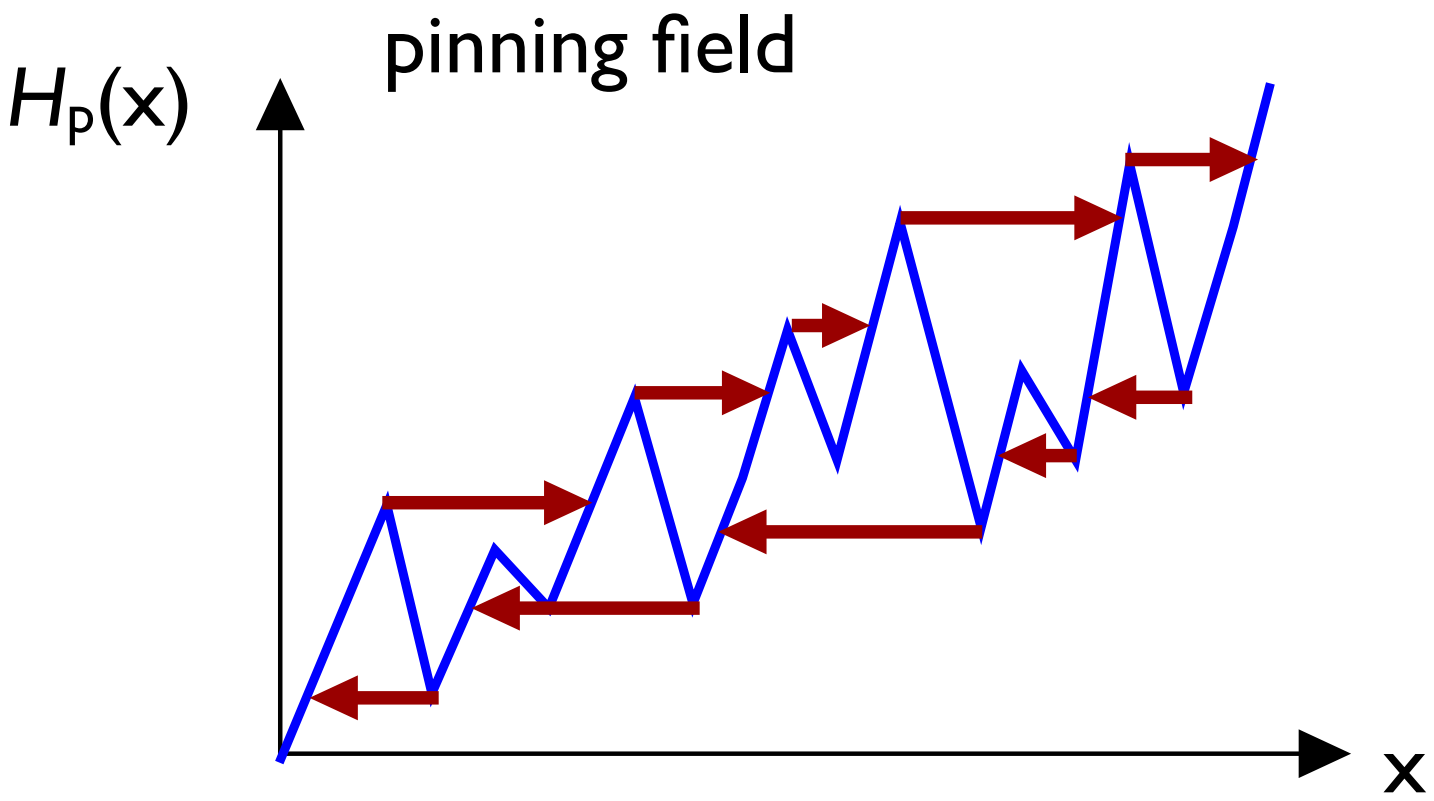
local minima

**irreversibility**

energy dissipation

# Hysteresis loop

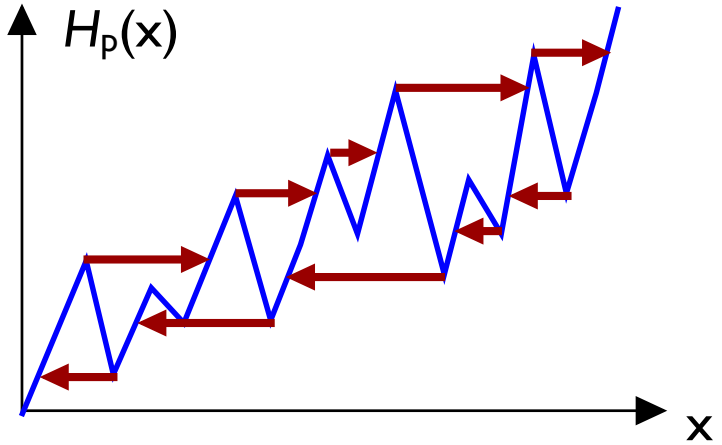
stochastic process



# Hysteresis loop

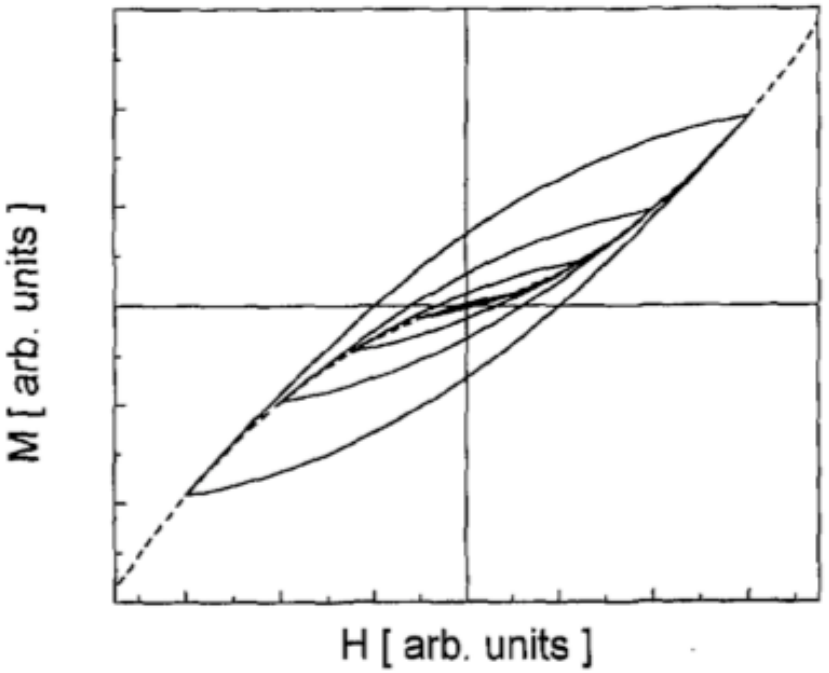
1) pinning field as a stochastic process

i.e. the Wiener process




2) take the average

Rayleigh loops



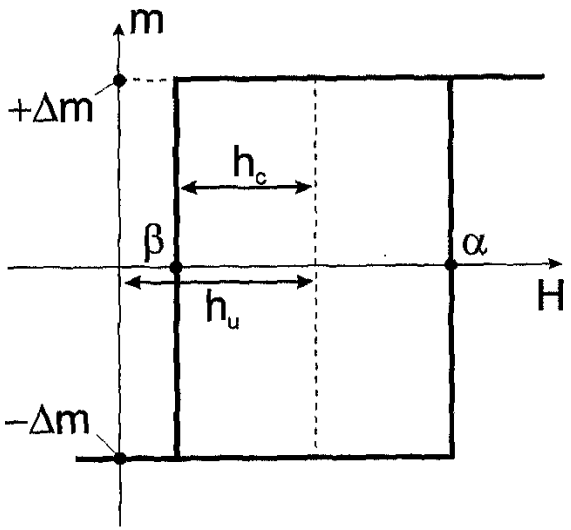

$$M \pm M_{max} = \chi_{in}(H_a \pm H_{max}) \pm \frac{b}{2}(H_a \pm H_{max})^2$$

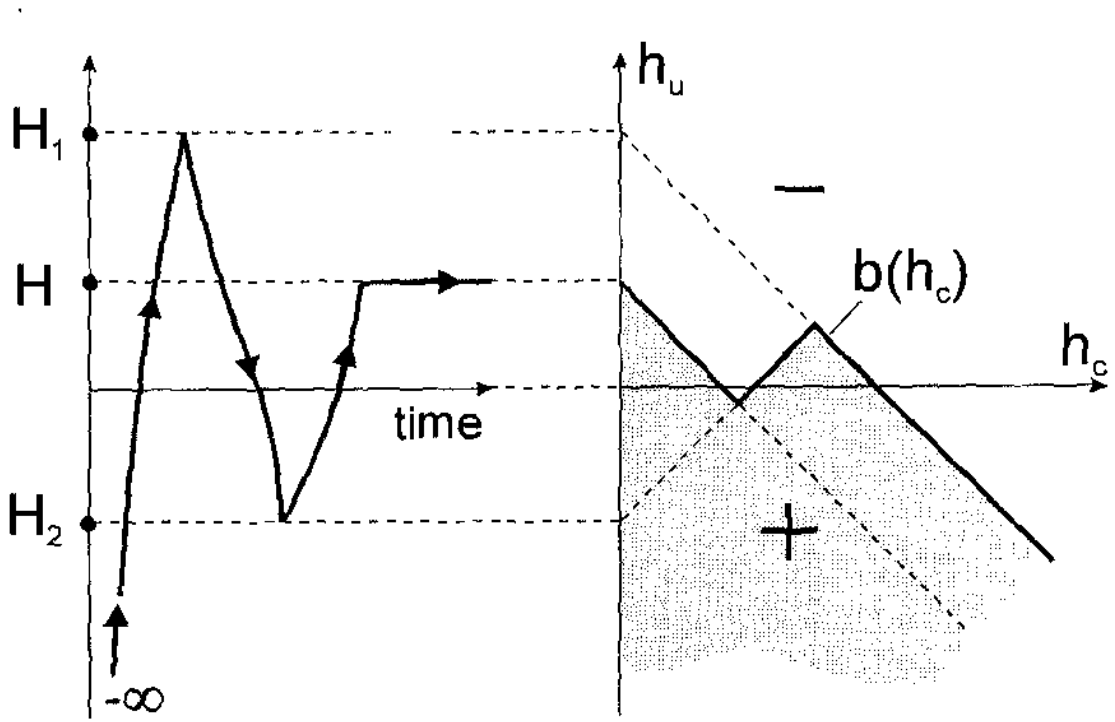


Louis Néel (1904-2000)

# Hysteresis loop

## The Preisach model of hysteresis




- Mayergoyz "representation theorem"

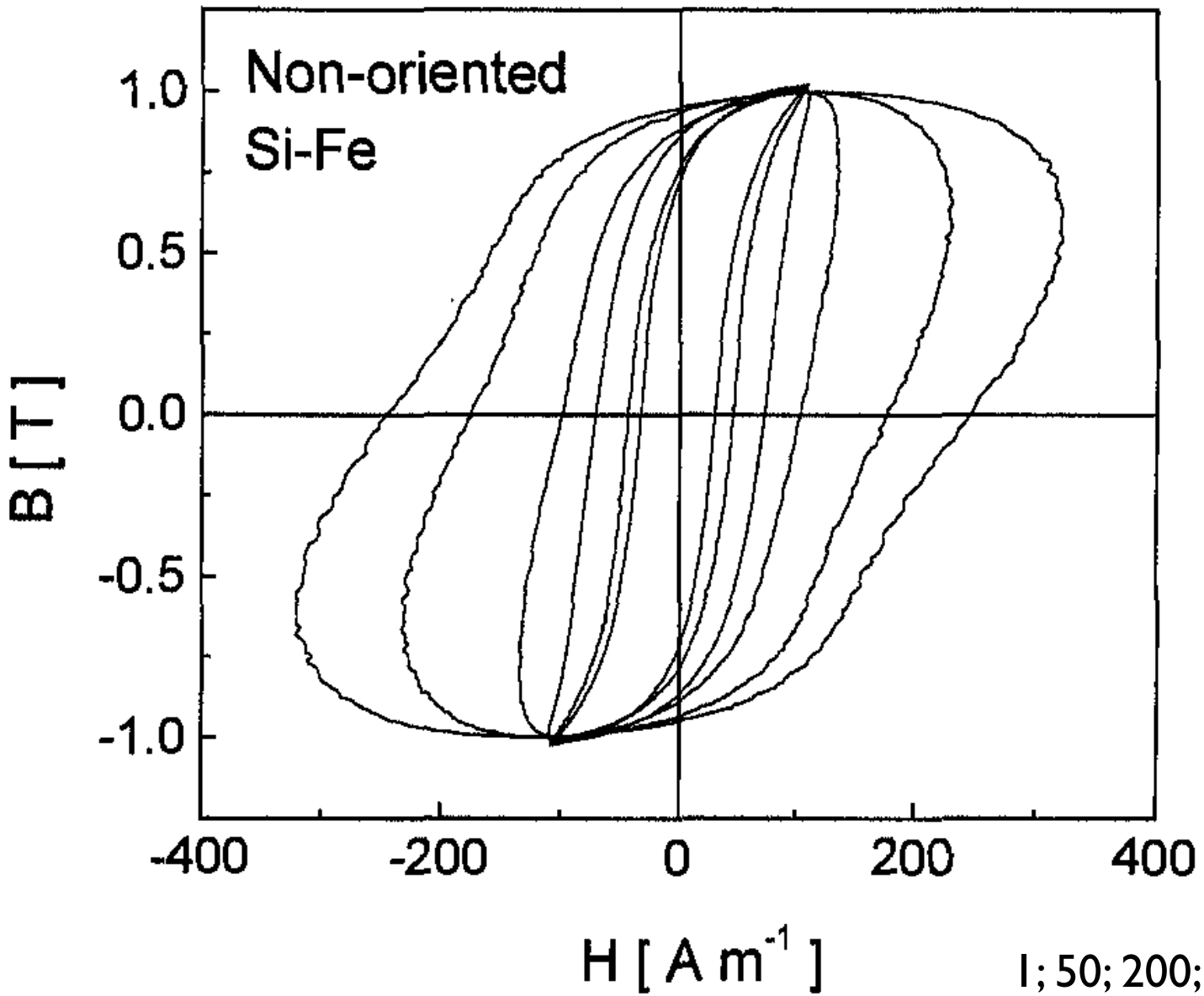
  necessary and sufficient conditions:
  - wiping out
  - minor loop congruency

## The dynamic hysteresis loop

- Eddy currents and the theory of losses

# Dynamic hysteresis loop

Frequency dependent




1; 50; 200; 400; 1k, 1.6k Hz

# Dynamic hysteresis loop

## *Eddy currents*

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

in metals

$$\mathbf{j} = \sigma_e \mathbf{E}$$

***Eddy current field***

$$\begin{aligned} \nabla \cdot \mathbf{H}_{eddy} &= 0 \\ \nabla \times \mathbf{H}_{eddy} &= \mathbf{j}_{eddy} \end{aligned}$$

magnetic polarization

$$\mathbf{J} = \mu_0 \mathbf{M}$$

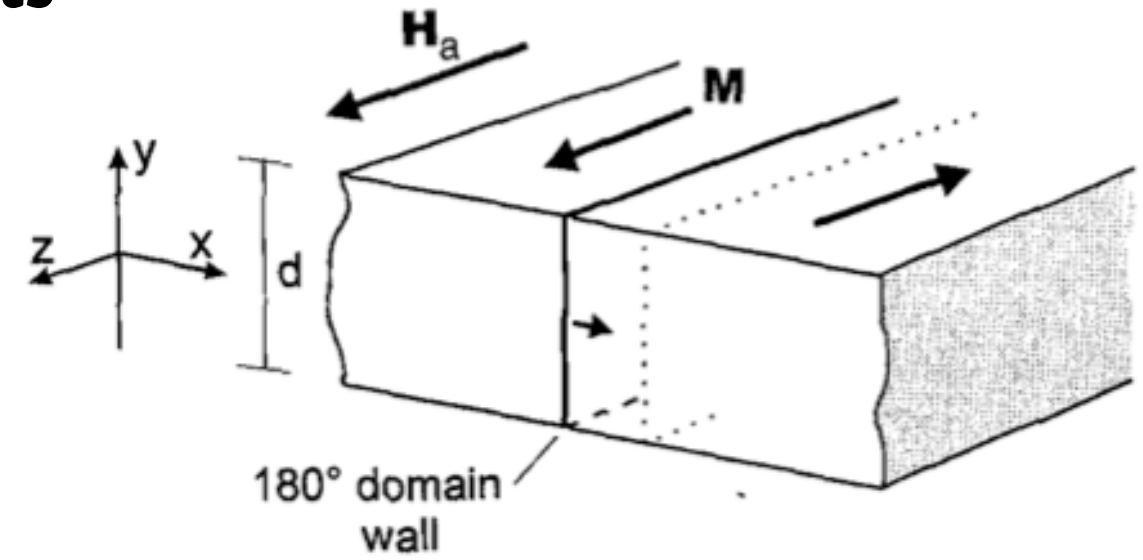
in soft magnetic materials

$$\mathbf{B} \simeq \mathbf{J}$$

# Domain wall motion

## *Eddy currents*

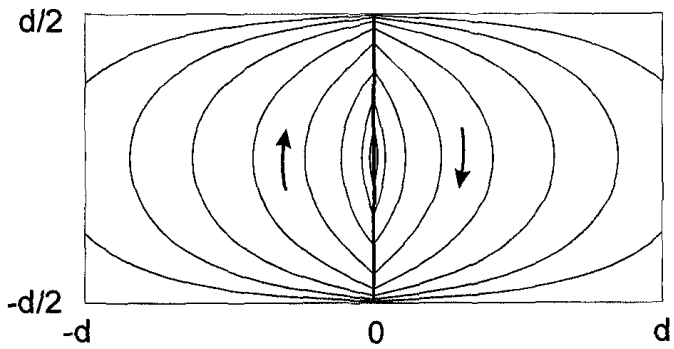
slab




$$\sigma_e G \frac{d\phi}{dt} = H_a - H_p(x)$$

*G*=0.1356

$\mathbf{j}_{eddy}$



# Theory of losses

*loss separation*

$$W = W_{hyst} + W_{cl} + W_{exc}.$$

hysteresis loss (domains)

classical loss (slab geometry)

excess loss (domains)

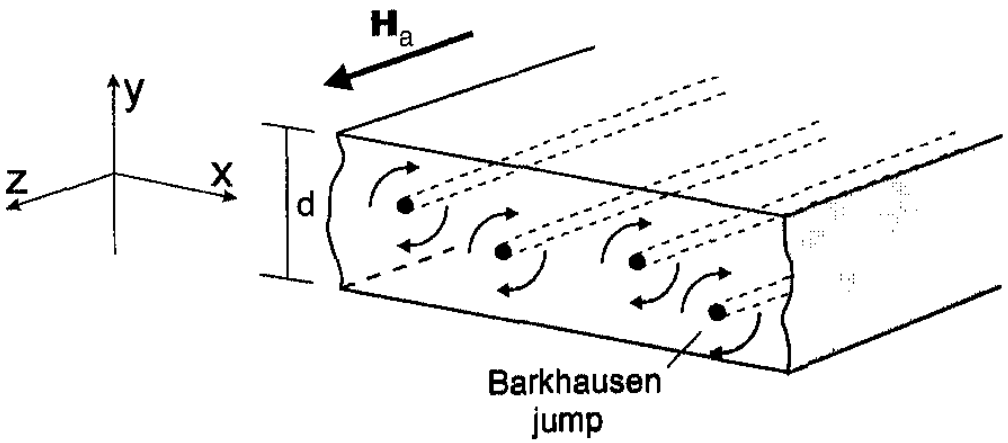
$$P = W \cdot f$$

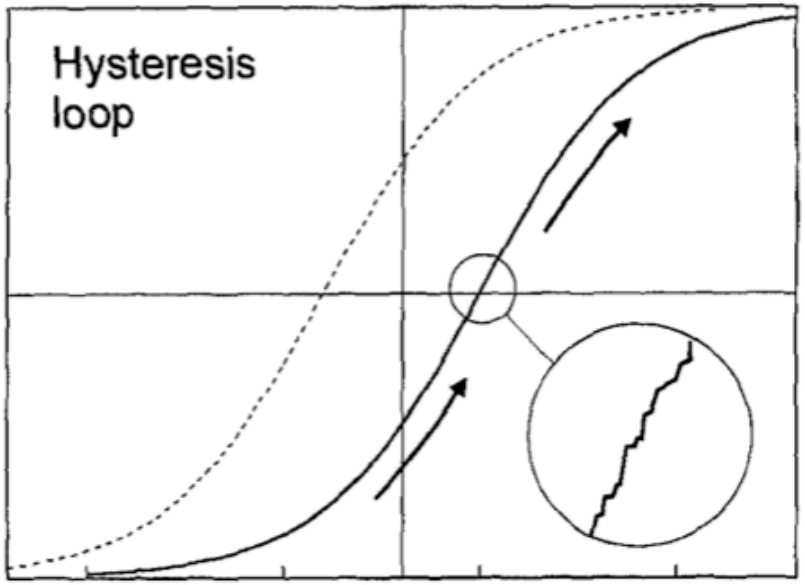
power loss

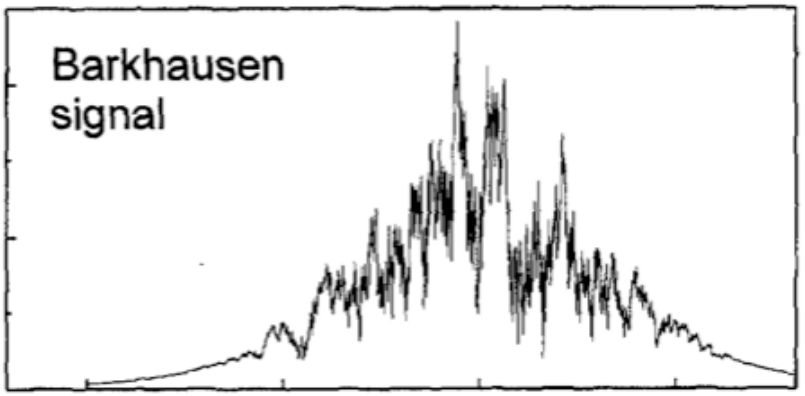
energy loss per cycle

# Theory of losses

hysteresis loss



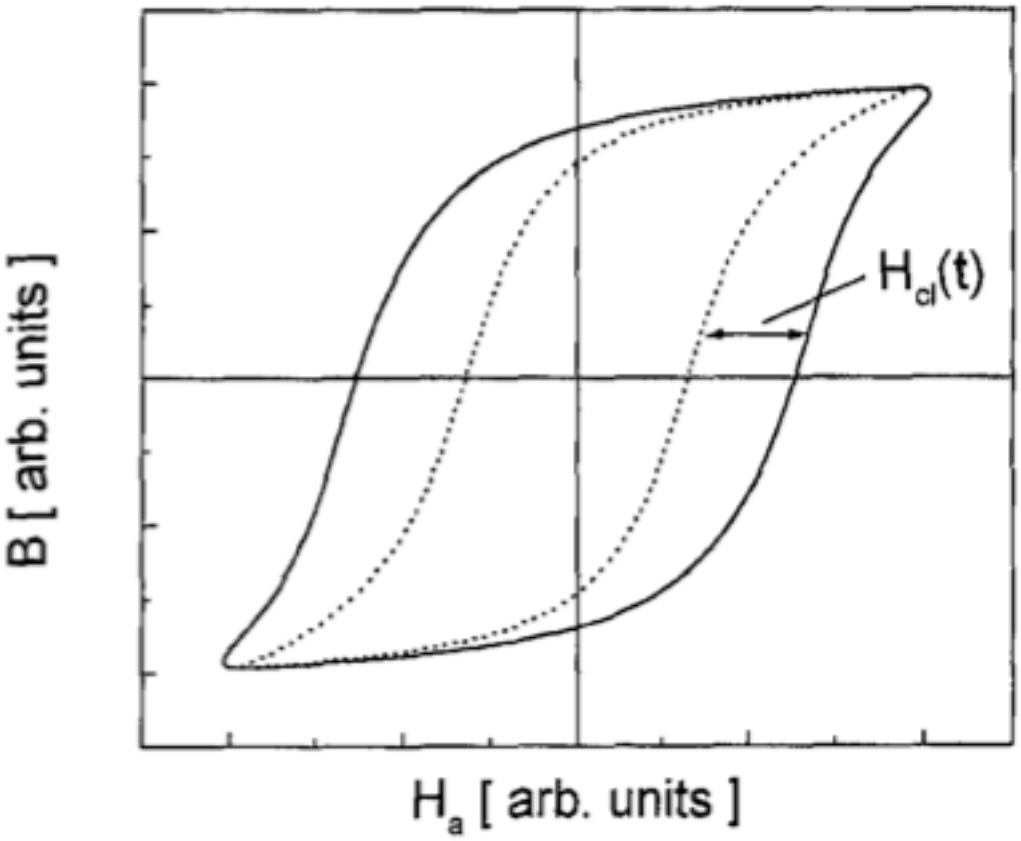



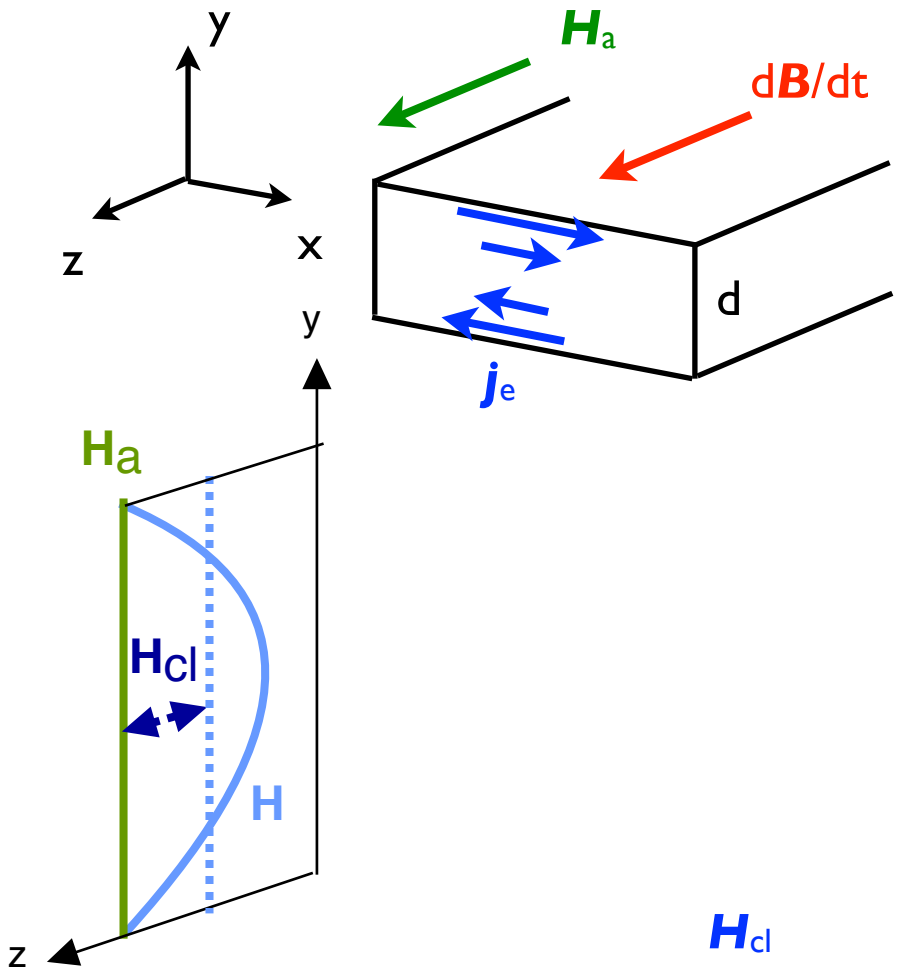




$$W_{hyst}(J_p)$$

# Theory of losses

classical loss



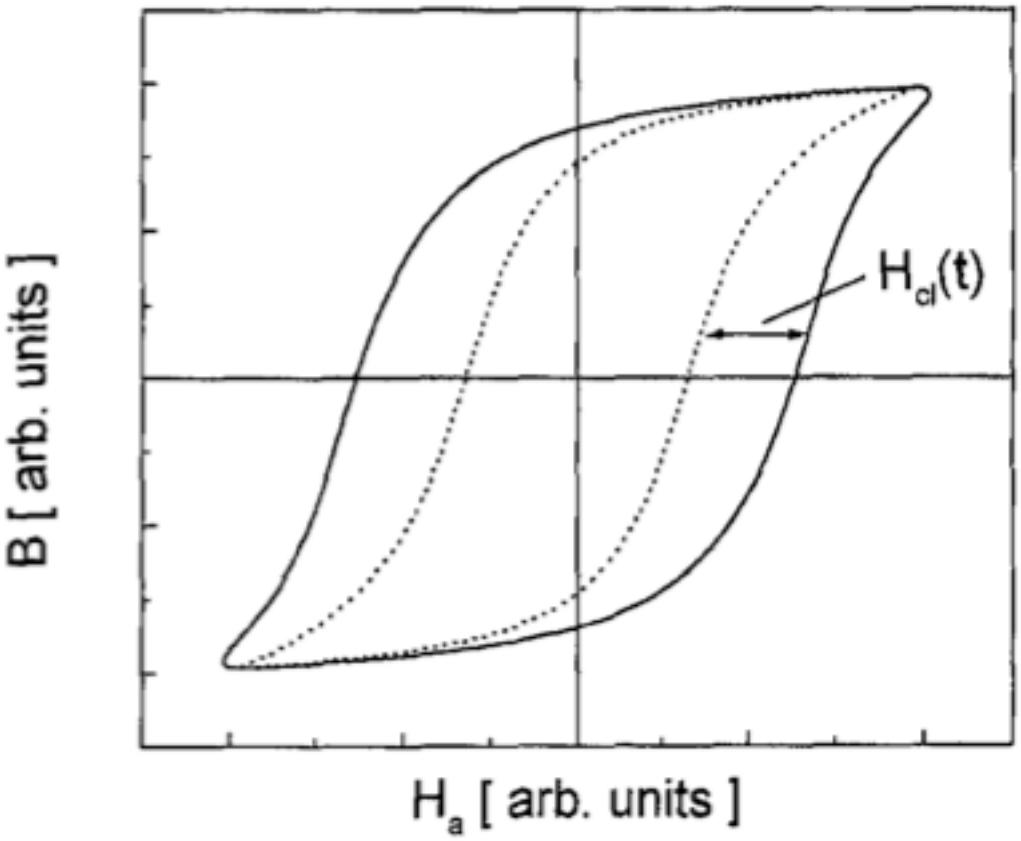

$$H(y,t) = H_a(t) - \frac{\sigma_e}{2}\frac{dJ}{dt}\left(\frac{d^2}{4} - y^2\right)$$

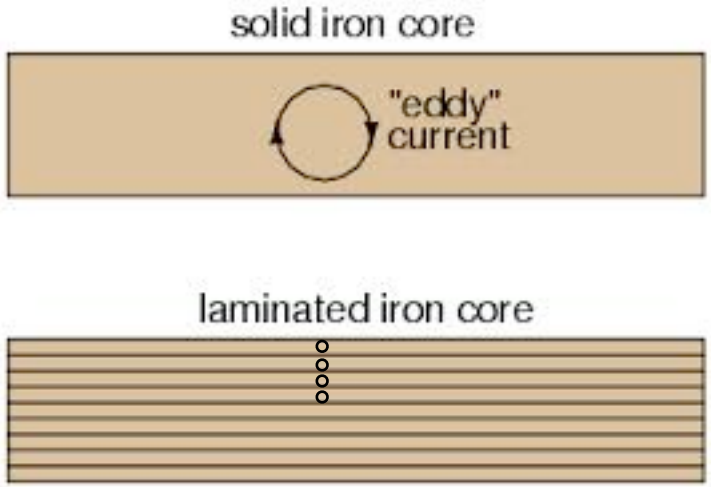
$H_{cl}$

$$< H(t) >_y = H_a(t) - \frac{\sigma_e d^2}{12}\frac{dJ}{dt}$$

# Theory of losses

classical loss



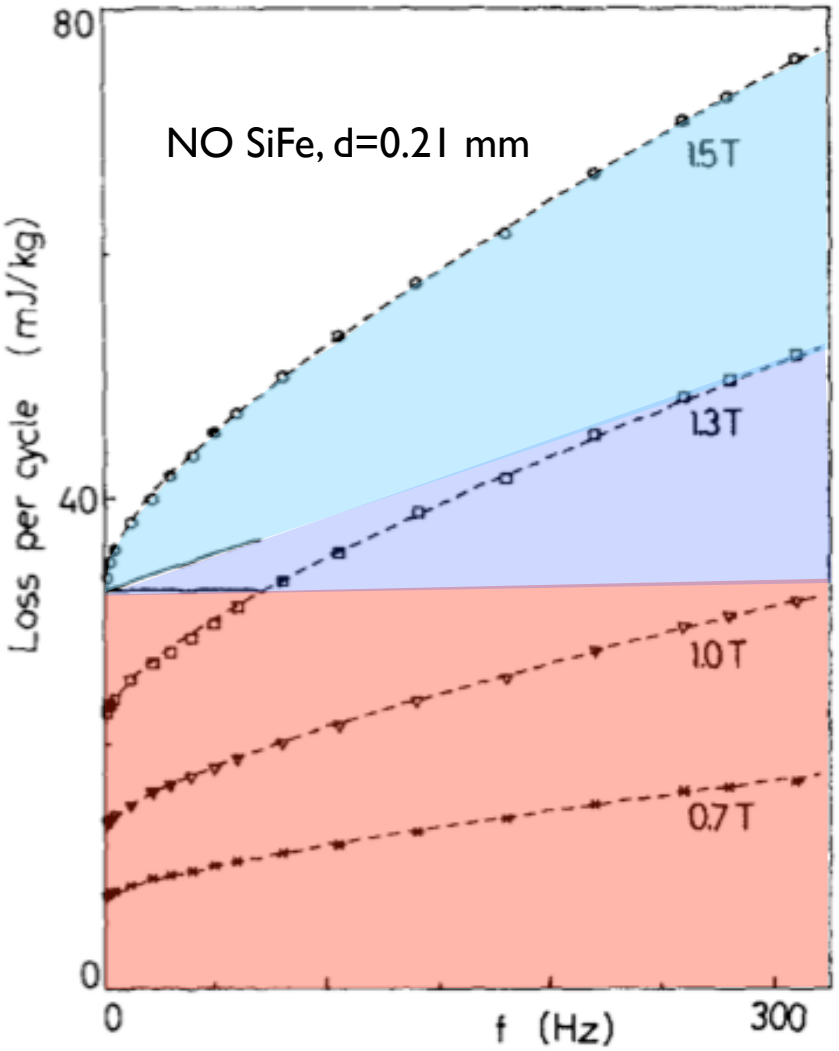




$$W = W_{hyst}(J_p) + \frac{\pi^2}{6}\sigma_e d^2 J_p^2 f$$

small conductivity

small thickness

# Theory of losses




excess loss

classical

hysteresis

$$W = W_{hyst}(J_p) + \frac{\pi^2}{6}\sigma_e d^2 J_p^2 f$$

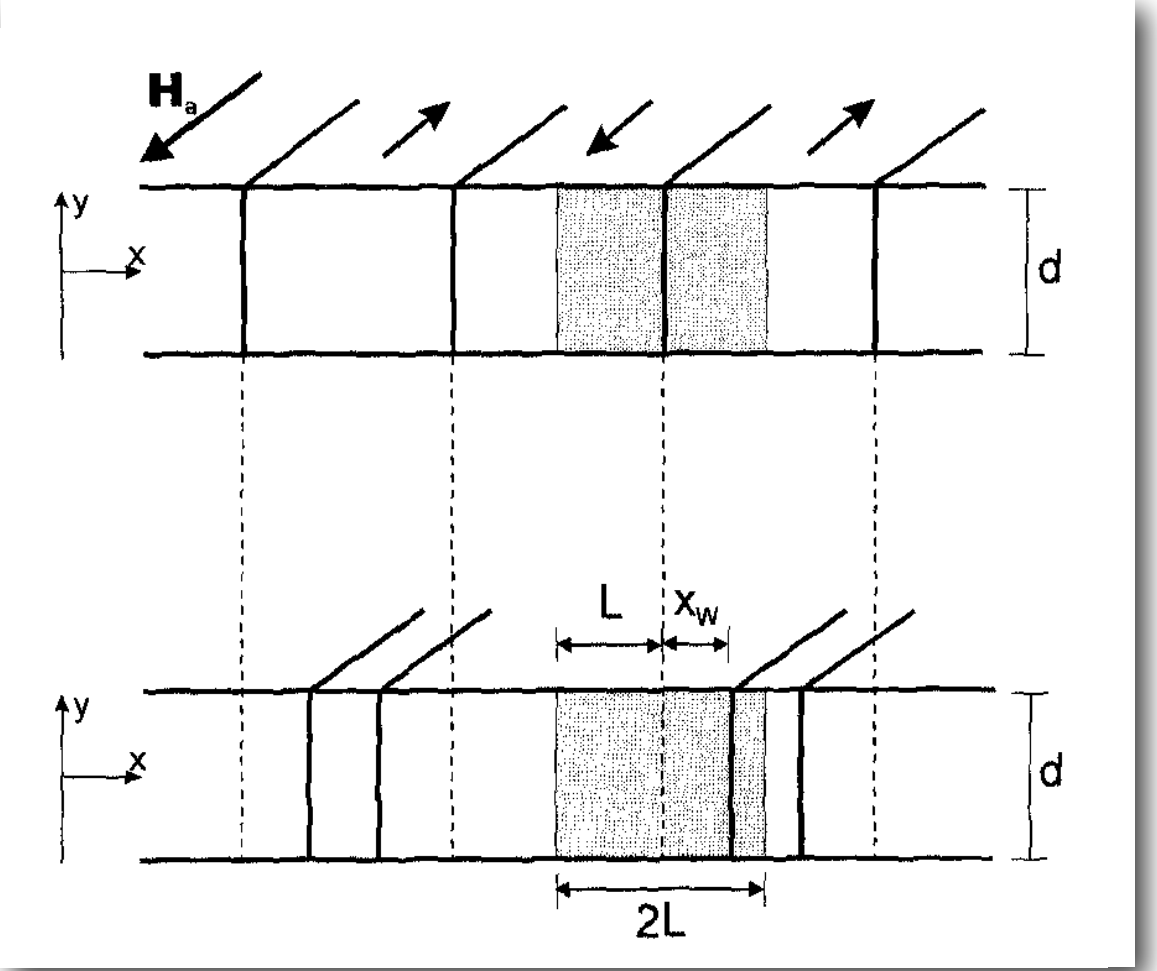
hysteresis classical

# Theory of losses

excess loss

*periodic domain structure*

*model of Pry and Bean*



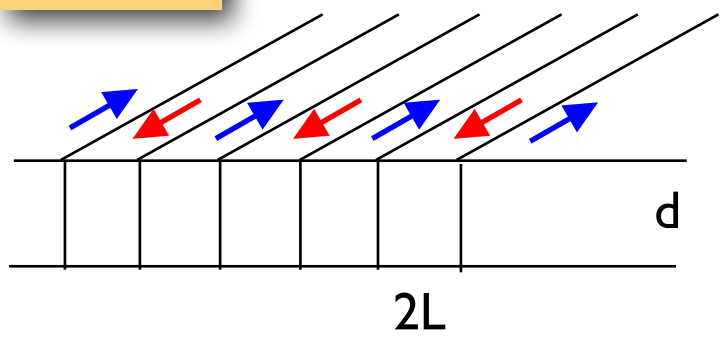
# Theory of losses

excess loss

*periodic domain structure*

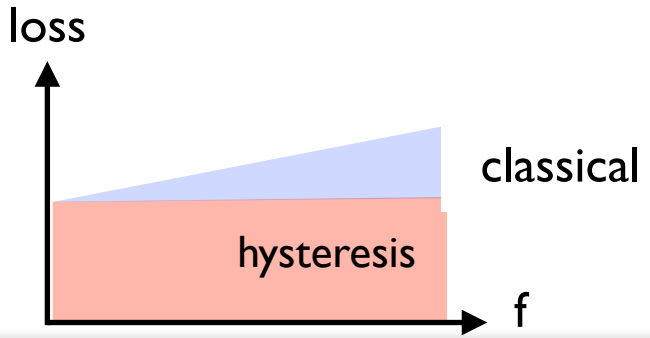
*model of Pry and Bean*

2L≪d




$$P = \frac{\sigma_e d^2}{12}\left(\frac{dJ}{dt}\right)^2$$

classical loss!



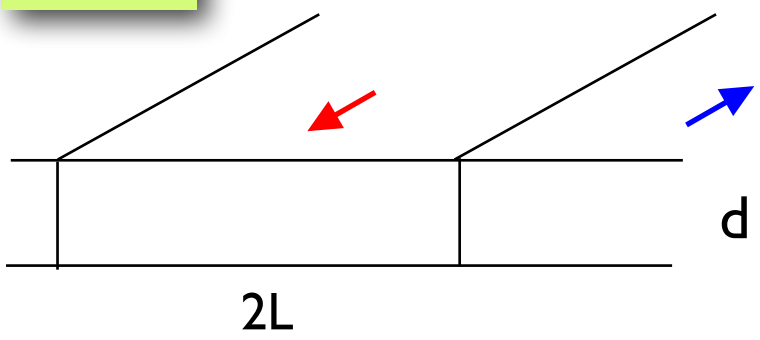
# Theory of losses

excess loss

*periodic domain structure*

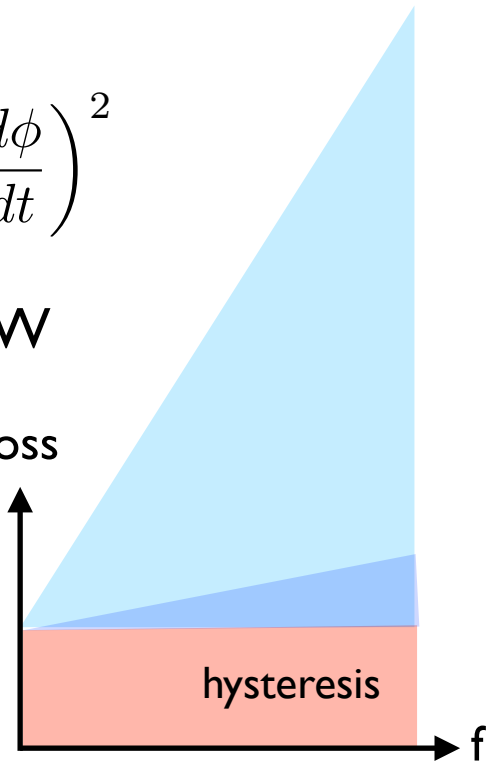
*model of Pry and Bean*

2L≫d

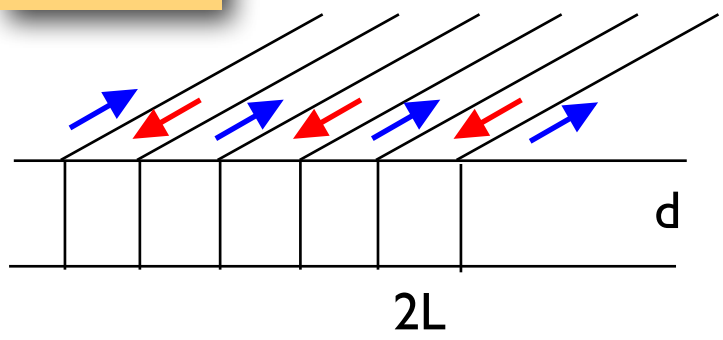



$$P = \sigma_e G \left(\frac{d\phi}{dt}\right)^2$$

individual DW

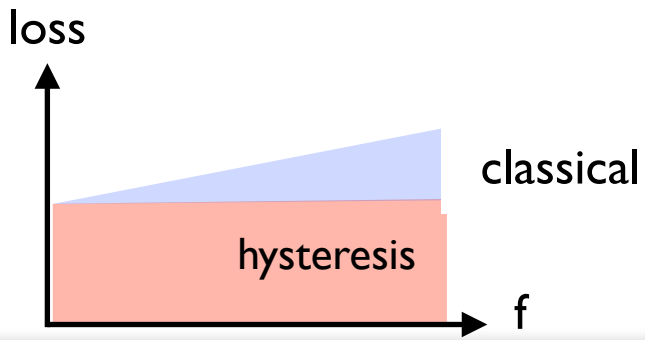



2L≪d




$$P = \frac{\sigma_e d^2}{12} \left(\frac{dJ}{dt}\right)^2$$

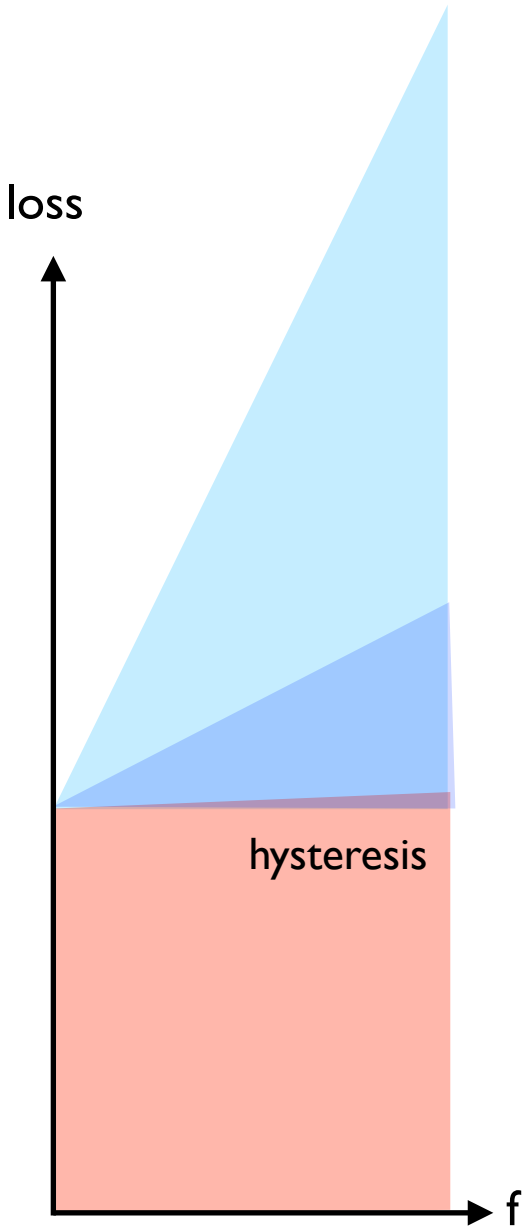
classical loss!

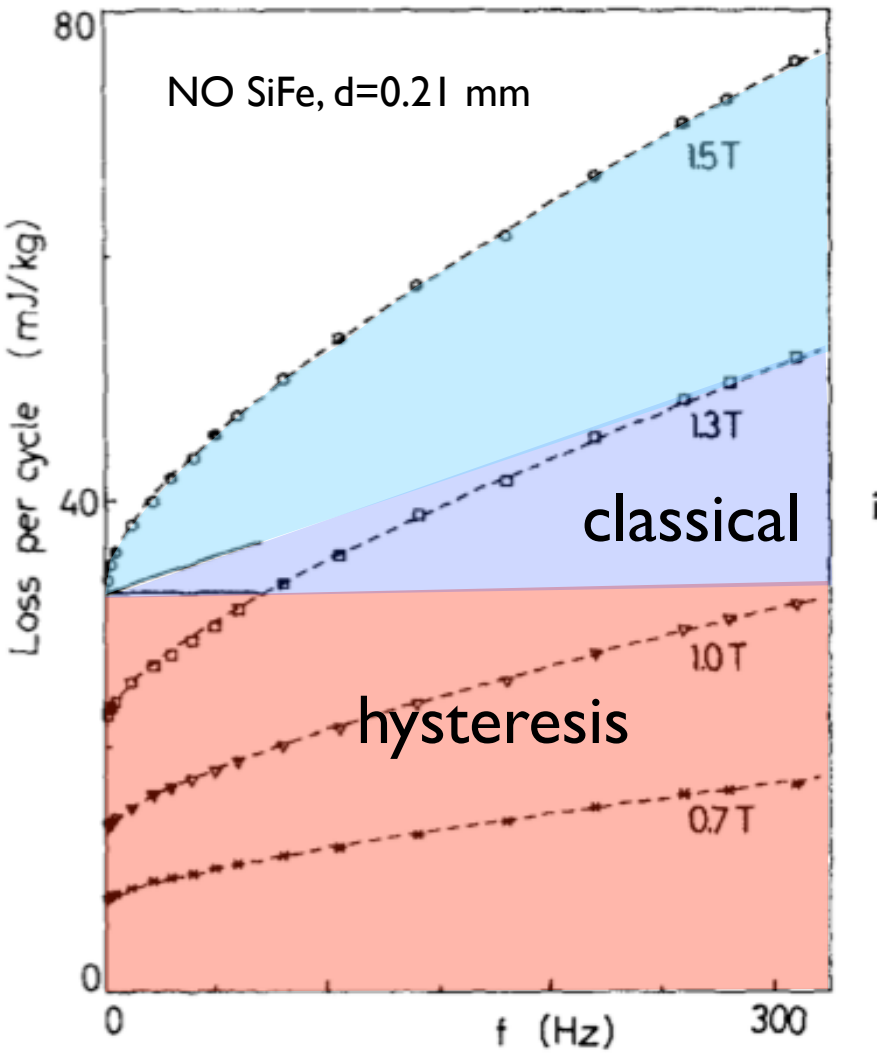


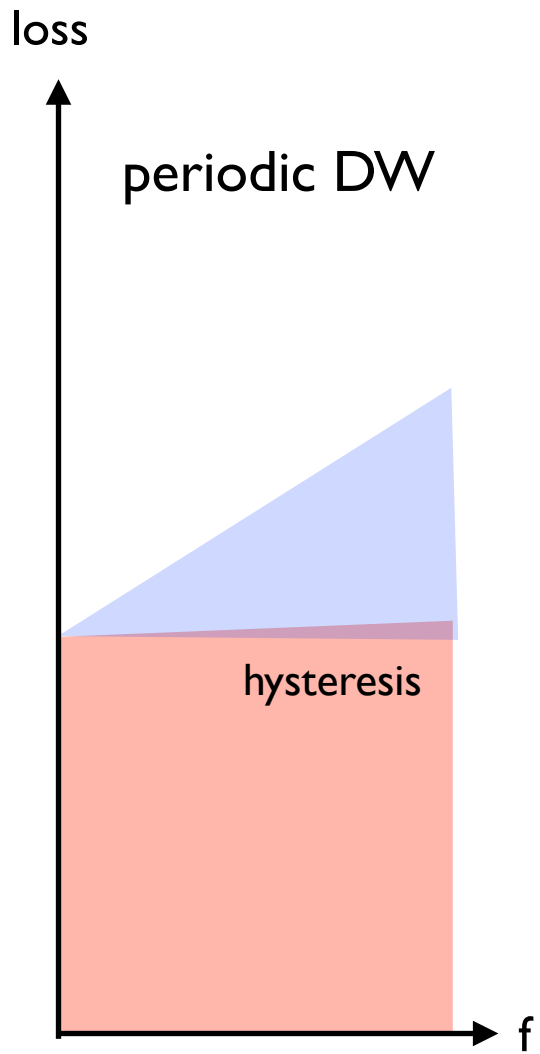
# Theory of losses

individual DW

is the number of DWs proportional to the frequency?



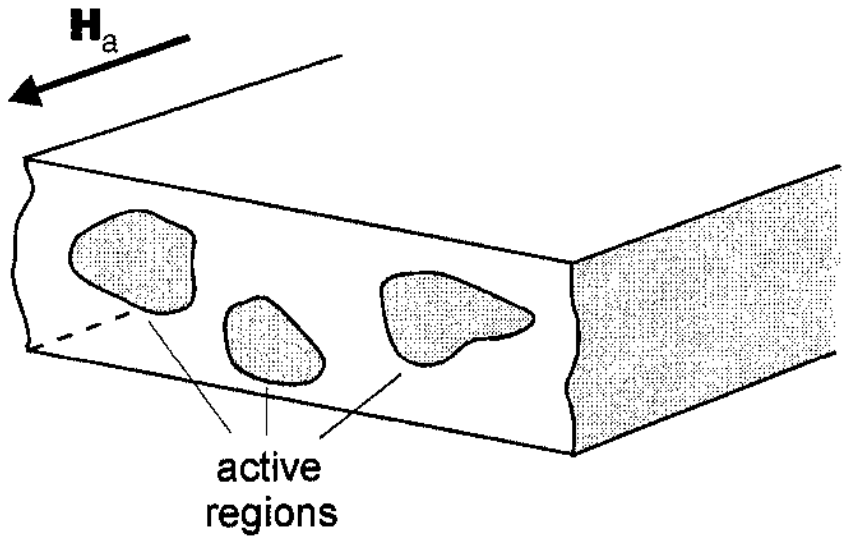



# Theory of losses

## excess loss

1) excess field for a single DW

$$H_{DW} = \sigma_e G S \frac{dJ}{dt}$$

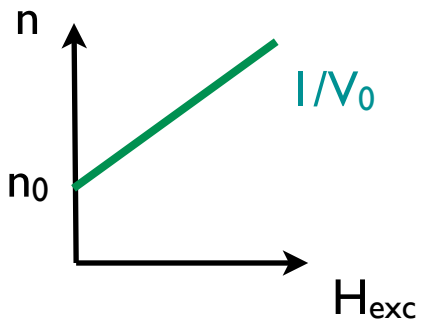



2) excess field for *n* DWs

$$H_{exc} = \frac{H_{DW}}{n}$$

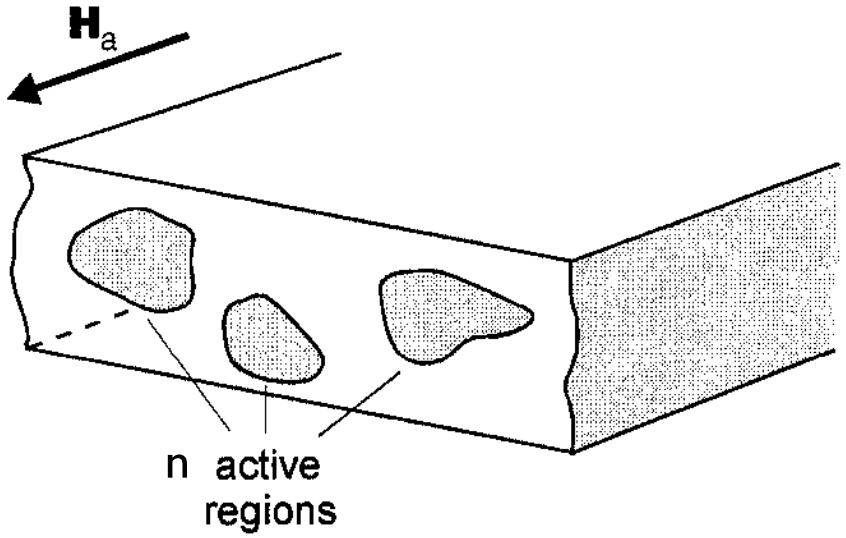
3) *n* DWs proportional to the excess field

$$n(H_{exc}) = n_0 + \frac{H_{exc}}{V_0} + \ldots$$

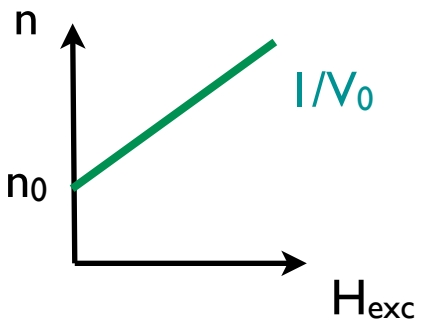


# Theory of losses

excess loss



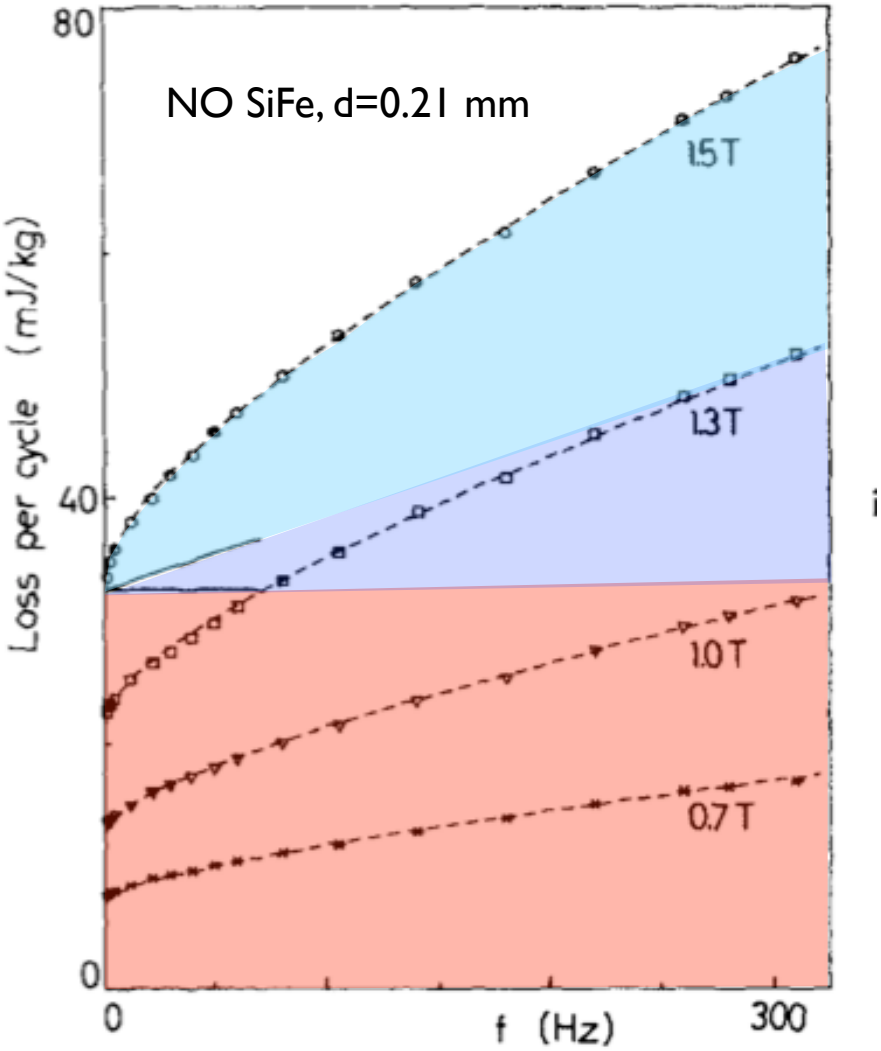

$$H_{exc} = \frac{n_0 V_0}{2}\left(\sqrt{1+\frac{4\sigma_e G S}{n_0^2 V_0}\frac{dJ}{dt}}-1\right)$$

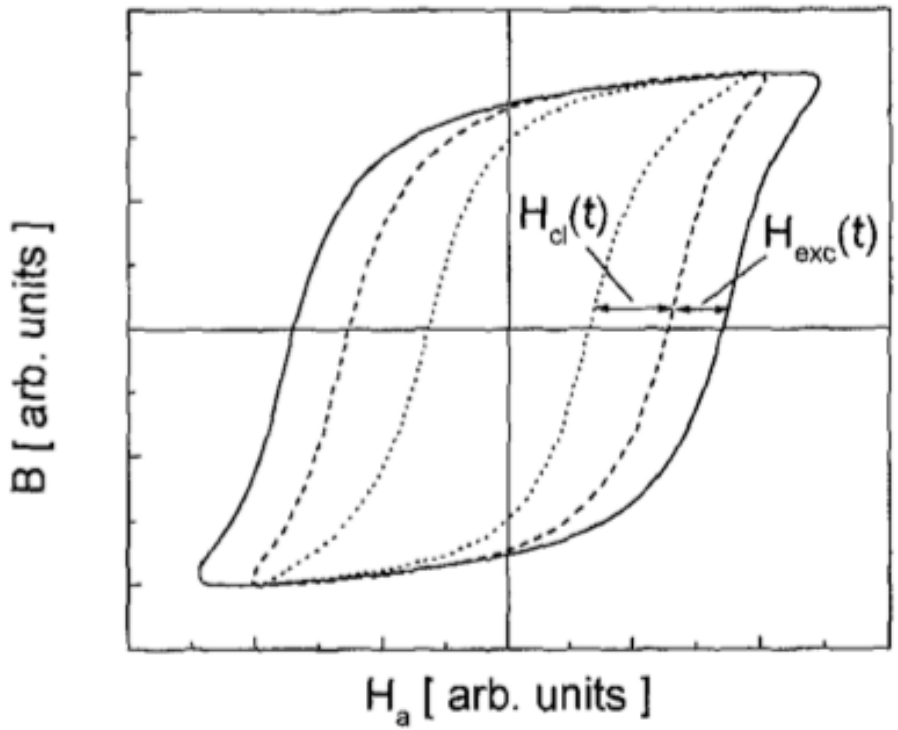



with $n_0$ small

$$W = W_{hyst}(J_p) + \frac{\pi^2}{6}\sigma_e d^2 J_p^2 f + 8\sqrt{\sigma_e G S V_0}\, J_p^{3/2} f^{1/2}$$

# Theory of losses




$$W = W_{hyst}(J_p) + \frac{\pi^2}{6}\sigma_e d^2 J_p^2 f + 8\sqrt{\sigma_e G S V_0}\, J_p^{3/2} f^{1/2}$$

hysteresis classical excess

**thanks for your attention !**

# Bibliography

G. Bertotti, *Hysteresis in Magnetism*. Academic Press, 1998.
F. Fiorillo, *Characterization and measurement of magnetic materials*, Academic Press, 2004.