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Modelling and analysis of silicon spin qubits

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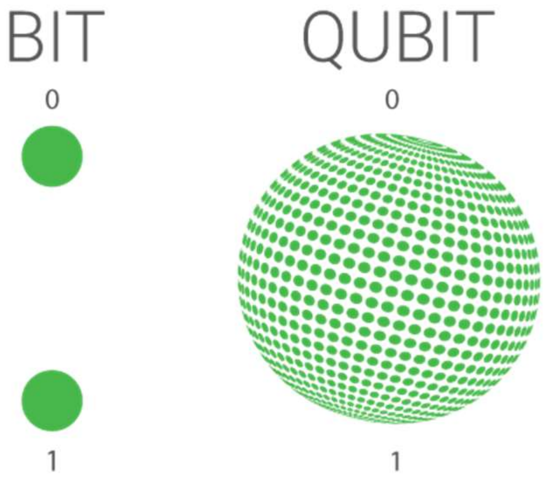
Outline

- ❑ Introduction: basic principle and motivations
- ❑ Quantum dot based qubit:
- • Loss-DiVincenzo qubit
- • Singlet-triplet qubit
- • Exchange only qubit - Hybrid qubit
- ❑ Donor based qubit:
- • Donor bounded e^- qubit
- ❑ Quantum dot-Donor based qubit:
- • $e^-/^{31}P^+$ flip-flop qubit
- • Carroll qubit
- ❑ Hybrid qubit: effective Hamiltonian model and derivation of single and double quantum operations.
- ❑ Conclusions

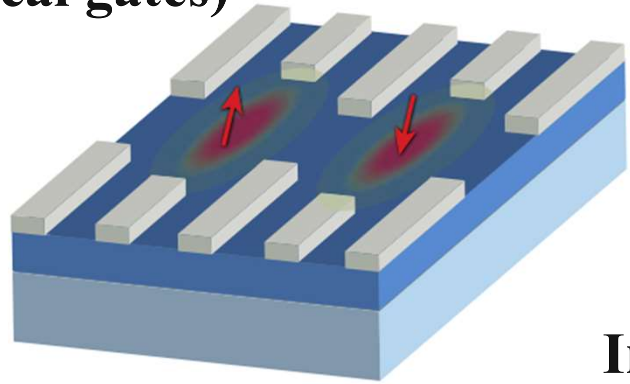
DiVincenzo's criteria



- Scalable physical system with well characterized qubit
- Ability to initialize the state of the qubits to a simple fiducial state, i.e. $|0000...\rangle$
- Long relevant coherence times (larger than gate operations)
- A universal set of quantum gates: i.e. (H, $\pi/8$, $R_Z(\pi/4)$, c-NOT)
- A qubit-specific measurement capability, i.e. single quantum measurement

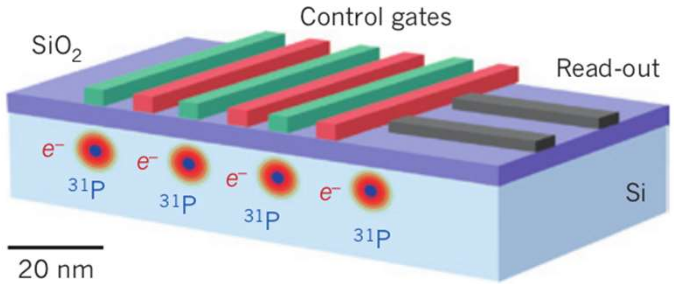
Silicon spin qubits: confinement of electrons

Quantum dot
(by means of heterostructures or electrical gates)



Long coherence time
Fast gate operations
Easy manipulation
Potential for scaling
Nuclear free isotopes
Integrability with CMOS infrastructure

Cryogenic temperature:
The thermal energy ($k_B T$) must be below charging energy ($e^2/2C$)



Donor
(group V atoms for Silicon)

J. J. L. Morton et al., Nature 479, 345–353 (2011)

Qubit types

Table 1. Qubit types grouped by the number of spin-1/2 per qubit, kind of qubit (physical or virtual) and the type of confining element: QD, donor or QD-donor.

Number of spins	Qubit type	QD-based	Donor-based	QD-donor -based
1	Physical	Loss-DiVincenzo [17]	donor bounded e^- [18]	
2	Virtual	singlet-triplet [19, 20]		$e^-/^{31}P^+$ flip-flop [21, 22] Carroll [23–25]
3	Virtual	hybrid [26, 27] exchange only [1]		

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https://doi.org/10.1088/1361-6463/acd8c7

Topical Review

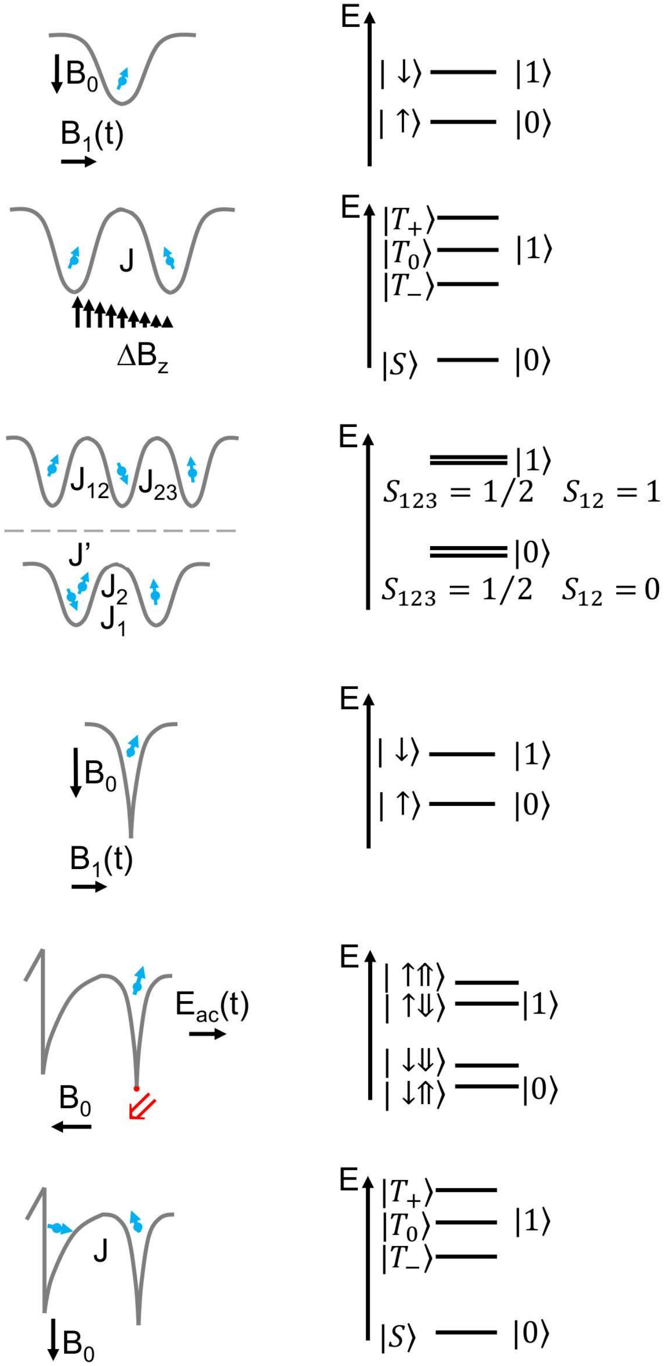
Silicon spin qubits from laboratory to industry

Marco De Michielis[1,*], Elena Ferraro[1,*], Enrico Prati[2], Louis Hutin[3], Benoit Bertrand[3], Edoardo Charbon[4], David J Ibberson[5] and Miguel Fernando Gonzalez-Zalba[5]

Qubit confining profiles and their corresponding energy level diagram

Qubit type | Level diagram

- **Loss-DiVincenzo qubit** a)
- **Singlet-triplet qubit** b)
- **Exchange-only qubit** c)
- **Hybrid qubit** d)
- **Donor bounded e⁻ qubit** e)
- **$e^-/^{31}P^+$ flip-flop qubit** f)
- **Carroll qubit** g)



Figures of merit

T_2^*	Coherence Time ([1])
T_{gate}	Gate Time
T_{init}	Initialization Time
T_{meas}	Measurement Time
F_{1Q}	1 Qubit Gate Fidelity ([2])
F_{2Q}	2 Qubit Gate Fidelity
F_{init}	Initialization Fidelity
F_{meas}	Measurement Fidelity

([1]) Extracted from a Ramsey experiment represents the rate of decay toward a mixed state, when the qubit is initialized to the state |1>. It is the dephasing time or the transverse relaxation time of the qubit on the Bloch sphere as a result of both energy relaxation and pure dephasing in the transverse plane. Unlike T_2, which is measured by Hahn experiment, is sensitive to inhomogenous broadening.
([2]) How "close" are the obtained gates from the ideal gates.

Spin qubit theory

Hubbard-like model:
Kinetic+Potential Energies

Effective Spin Hamiltonian:
only spin operators

Physical logical gates:
the controls are the electrical and/or the magnetic knobs.
Analytical or numerical calculations.

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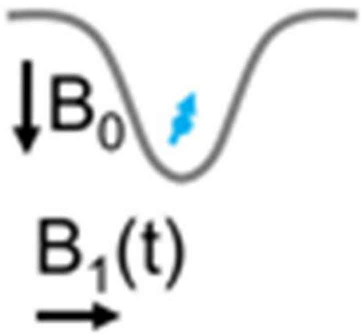
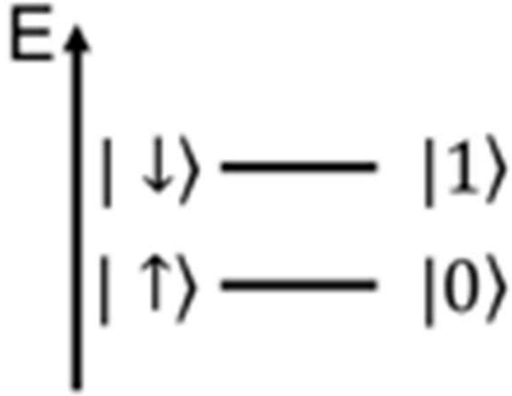
Loss-DiVincenzo qubit

1 spin confined in 1 QD





$$H=\frac{\hbar}{2}\omega_z\sigma_z+\hbar\Omega_x\cos(\omega t)\sigma_x$$

$$\hbar\omega_z = g_e\mu_B B_0 \qquad \hbar\Omega_x = g_e\mu_B B_1/2$$

$$S = 1/2, S_z = \pm 1/2$$

$$|0\rangle \equiv |\uparrow\rangle$$

$$|1\rangle \equiv |\downarrow\rangle$$

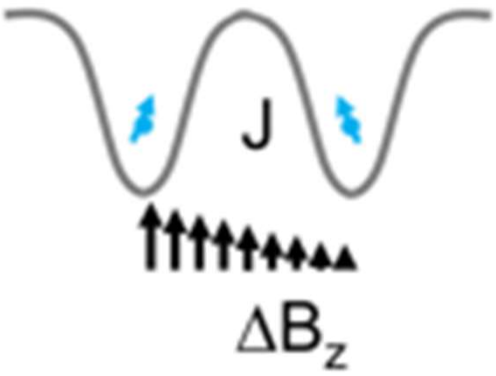
- **Universal qubit control through magnetic field pulses**
  - Via electron spin resonance (ESR) using local AC magnetic fields
  - Via local manipulation in a global magnetic fields

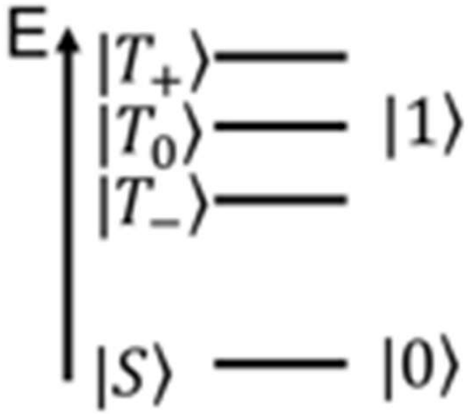
- **All electrical manipulation via AC electric fields in a magnetic field gradient.**

- **Long coherence times of the order of milliseconds.**

Singlet-Triplet qubit

2 spins confined in 2 QDs





$$H = \frac{\hbar}{2}\Delta\omega_z(\sigma_1^z - \sigma_2^z) + \frac{1}{4}J\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$J = \frac{4(t - J_t)^2}{U - U' - |\Delta\varepsilon|} - 2J_e$$

$$S_z = 0$$

$$|0\rangle \equiv |S\rangle$$

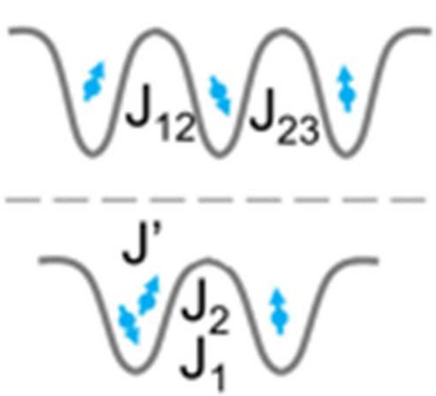
$$|1\rangle \equiv |T_0\rangle$$

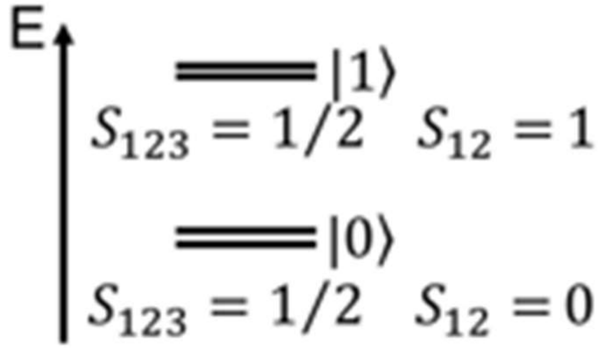
- **Spin rotations are performed by manipulating the exchange coupling J and a magnetic field gradient ΔE_z between the quantum dot.**
- **Fast readout and fast manipulation.**
- **Challenge: creation of local magnetic gradient**

✘Strategy: use of micromagnet in close proximity

Exchange only and Hybrid qubit

3 spins confined in 3 QDs or in 2 QDs





$$|0\rangle \equiv |S\rangle|\uparrow\rangle$$

$$S = 1/2, S_z = 1/2 \qquad |1\rangle \equiv \sqrt{\tfrac{1}{3}}|T_0\rangle|\uparrow\rangle - \sqrt{\tfrac{2}{3}}|T_+\rangle|\downarrow\rangle$$

$$H = \frac{1}{4}J_{12}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{4}J_{23}\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 + \frac{1}{4}J_{13}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3$$

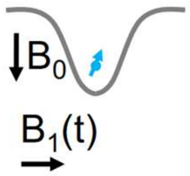
In a linear arrangement J_{13}=0

$$H = \frac{\hbar}{2}\omega_z(\sigma_1^z + \sigma_2^z + \sigma_3^z) + \frac{1}{4}J'\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{4}J_1\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 + \frac{1}{4}J_2\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3$$

- **All electrical fast control via gate electrodes.**
- **Coherence times of the order of hundreds of nanoseconds.**

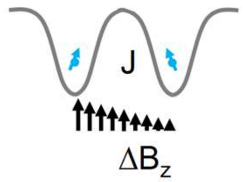
State of the art performances QD based qubit

Loss-DiVincenzo qubit



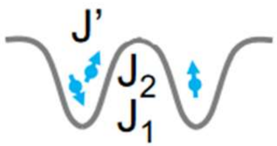
T_2^*	$^{28}Si/SiO_2$	120 µs
T_{gate}	Si/SiO_2	42 ns
T_{init}	Si/SiGe	50 µs
T_{meas}	Si/SiGe	24 µs
F_{1Q}	$^{28}Si/SiO_2$	99.957 %
F_{2Q}	$^{28}Si/SiGe$	99.81 %
F_{init}	Si/SiGe	99 %
F_{meas}	$^{28}Si/SiGe$	99 %

Singlet-Triplet qubit



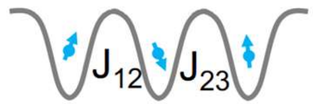
T_2^*	$^{28}Si/SiO_2$	12 µs
T_{gate}	$^{28}Si/SiO_2$	2.5 ns
T_{init}	Si/SiO_2	20 µs
T_{meas}	Si/SiO_2	50 ns
F_{1Q}	$^{28}Si/SiO_2$	99.76 %
F_{meas}	Si/SiO_2	75 %

Hybrid qubit



T_2^*	Si/SiGe	177 ns
T_{gate}	Si/SiGe	45 ns
F_{1Q}	Si/SiGe	94.5 %

Exchange-only qubit



T_2^*	$^{28}Si/SiGe$	2 µs
T_{gate}	$^{28}Si/SiGe$	10 ns
T_{meas}	$^{28}Si/SiGe$	0.98 µs
F_{1Q}	Si/SiGe	99.88 %
F_{meas}	$^{28}Si/SiGe$	99.975 %

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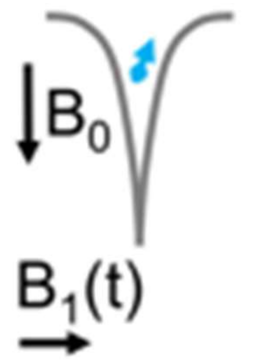
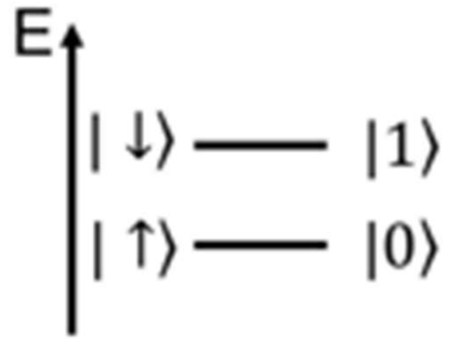
Donor-bounded e⁻ qubit

1 spin of an electron bound to a donor





$$H = \gamma_e B_0 S_z - \gamma_n B_0 I_z + \Omega_x \cos(\omega t) S_x + A \boldsymbol{S} \cdot \boldsymbol{I}.$$

In the high field limit ($\gamma_e B_0 \gg A$), the diagonal terms of the hyperfine interaction become negligible and the ESR allowed transition are confined in two distinct subspaces: the nuclear spin has down projection $|\Downarrow\rangle$ and up projection $|\Uparrow\rangle$.

$$H_{\{\uparrow,\downarrow\}\otimes\Downarrow} = \frac{\hbar}{2}(\omega_{12} - \omega)\sigma_z + \frac{\hbar}{2}\Omega_x \sigma_x$$

$$H_{\{\uparrow,\downarrow\}\otimes\Uparrow} = \frac{\hbar}{2}(\omega_{34} - \omega)\sigma_z + \frac{\hbar}{2}\Omega_x \sigma_x;$$

- **Coherence times longer due to the weak spin-orbit coupling.**
- **The control and manipulation using local AC magnetic fields.**
- **The spin read out uses the detection of tunnelling to a nearby reservoir, the so-called energy filtering or Elzerman readout.**

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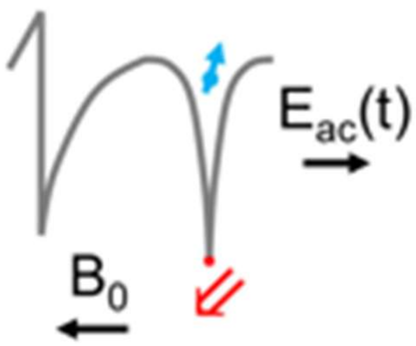
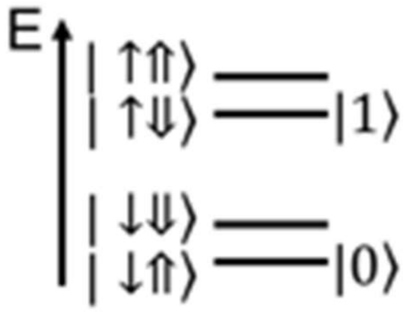
$e^-/{}^{31}P^+$ flip-flop qubit

^{31}P donor atom embedded in a ^{28}Si substrate at a depth *d* from the interface with a SiO_2 layer





$$H = H_{orb} + H_{B_0} + H_A$$

$$H_{orb} = -\frac{\varepsilon_0}{2}\sigma_z - \frac{deE_{ac}(t)}{2h}\left(\frac{de\Delta E_z}{h\varepsilon_0}\sigma_z + \frac{V_t}{\varepsilon_0}\sigma_x\right)$$

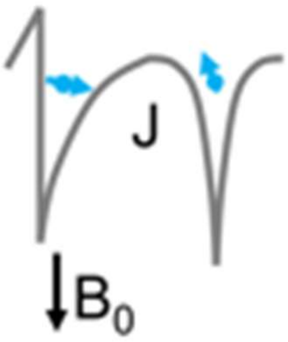
$$H_{B_0} = \gamma_e B_0\left[1 + \left(\frac{1}{2} + \frac{de\Delta E_z}{2h\varepsilon_0}\sigma_z + \frac{V_t}{2\varepsilon_0}\sigma_x\right)\Delta_\gamma\right]S_z - \gamma_n B_0 I_z$$

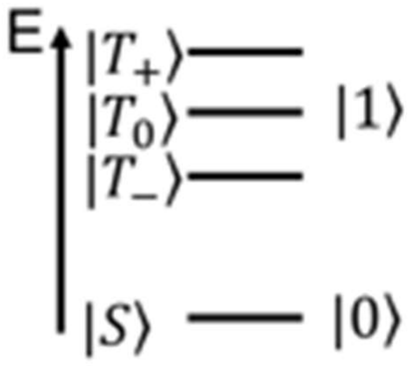
$$H_A = A\left(\frac{1}{2} - \frac{de\Delta E_z}{2h\varepsilon_0}\sigma_z - \frac{V_t}{2\varepsilon_0}\sigma_x\right)\mathbf{S}\cdot\mathbf{I},$$

- An electric dipole is created by the negatively charged electron and the positively charged donor nucleus. Taking advantage of the dipole-dipole interaction, it enables the coupling between two qubits up to distances of an order of magnitude higher than the Donor-bounded e^- qubit, in the 100-500 nm range.
- Electrically modulating the hyperfine interaction A by Ez causes an Electron Dipole Spin Resonance (EDSR) transition between the states with antiparallel spins, that are used to encode the qubit.

Carroll qubit

1 spin of an electron bound to a donor and 1 spin confined in a QD

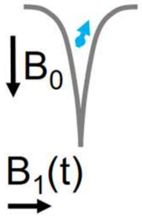




$$H = \gamma_e B_0 (S^z_{donor} + S^z_{dot}) - \gamma_n B_0 I_z + A\boldsymbol{S}_{donor} \cdot \boldsymbol{I} + J\boldsymbol{S}_{donor} \cdot \boldsymbol{S}_{dot}.$$

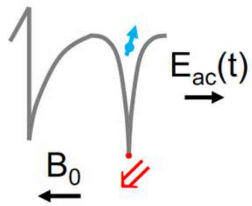
- Fast readout and fast manipulation via GHz one-axis electrical control.
- The logical states depend on the transition energies. For example, the qubit is defined by the singlet-triplet states between the electronic spin in the QD and the electronic spin in the donor.
- Qubit operations can be indeed performed on the donor electron spin and on the dot electron spin with a pulsed microwave field, which can be delivered locally or globally by placing devices into microwave cavities.

State of the art performances Donor and QD-Donor based qubit



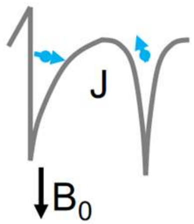
Donor-bounded e^- qubit

T_2*	^{28}Si:P	>100 μs
T_{gate}	^{28}Si:P	sub μs
F_{1Q}	^{28}Si:P	99.4 %



$e^-/^{31}P^+$ flip-flop qubit

T_2*	^{28}Si:P	4.09 μs
T_{gate}	^{28}Si:P	4.16 μs
F_{1Q}	^{28}Si:P	98.4 %



Carroll qubit

T_2*	^{28}Si:P	1.3 μs

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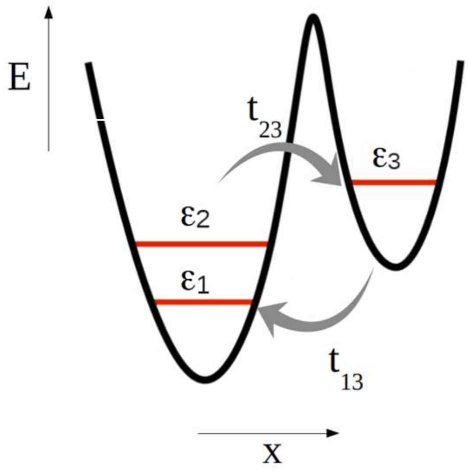
Hubbard-like model and Projection Operator method: Effective Hamiltonian model for the Hybrid Qubit



Total energy of the system:
Hubbard-like model

$$H = H_e + H_t + H_U + H_J$$

Single-electron energy level of each dot

Tunneling energy between the two dots

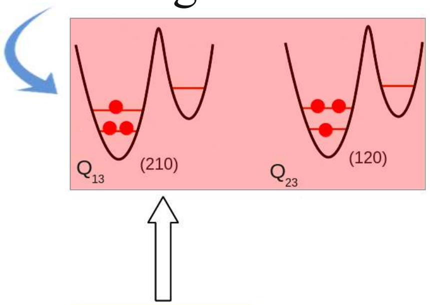
Coulomb and exchange interaction between pairs of electrons intra-dot and inter-dot

J.H. Jefferson, W. Hausler, Phys. Rev. B 54, 4936 (1996)

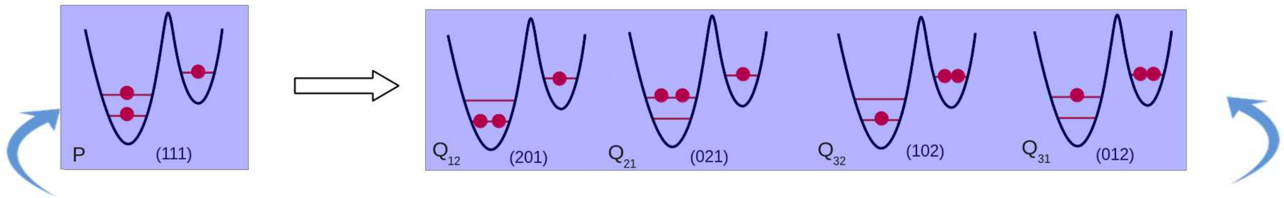
Projection Operator

The **total Hilbert space is divided** into subspace **P**, which consists of holes and single occupancies and subspace **Q** with at least one double occupancy.

Configurations that have higher energies and are neglected



All the possible configurations with single and double configurations arranged in three orbitals



Configuration of reference that correspond to the projection P

Configurations that contribute in a consistent way to the effective Hamiltonian

E. Ferraro, M. De Michielis, G. Mazzeo, M. Fanciulli, E. Prati, Quantum Information Processing 13, 1155-1173 (2014)

Effective Hamiltonian

SCHRÖDINGER EQUATION

$$H^{eff} \approx PHP - \sum_{i \neq j} \frac{1}{\Delta E_{ij}} PHQ_{ij}HP$$

ΔE_{ij} is the difference of the energy with one Q_{ij} and the energy of PHP, corresponding to the case in which there is only one electron in each orbital

$$H^{eff} \approx J_{13}\mathbf{S}_1 \cdot \mathbf{S}_3 + J_{23}\mathbf{S}_2 \cdot \mathbf{S}_3 + J_{12}\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$J_{13} \equiv J_1 \simeq \frac{1}{E_{(012)} - E_{(111)}} 4(t_{13} - J_t^{(13)})^2 - 2J_e^{(13)}$$

$$J_{23} \equiv J_2 \simeq \frac{1}{E_{(102)} - E_{(111)}} 4(t_{23} - J_t^{(23)})^2 - 2J_e^{(23)}$$

$$J_{12} \equiv J' = \left(\frac{1}{E_{(201)} - E_{(111)}} + \frac{1}{E_{(021)} - E_{(111)}} \right) 4J_t^{(12)2} - 2J_e^{(12)}$$

EFFECTIVE COUPLING CONSTANTS

Effective Hamiltonian

The three interacting electrons are used to encode a logical qubit

LOGICAL BASIS

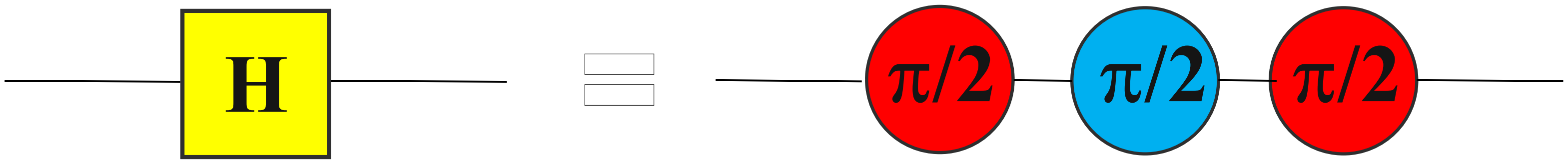
$$|0\rangle \equiv |S\rangle|\downarrow\rangle, \qquad |1\rangle \equiv \sqrt{\frac{1}{3}}|T_0\rangle|\downarrow\rangle - \sqrt{\frac{2}{3}}|T_-\rangle|\uparrow\rangle$$

$$|S\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}, \qquad |T_0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \qquad |T_-\rangle = |\downarrow\downarrow\rangle.$$

$$H = \begin{pmatrix} -\frac{E_z}{2} - \frac{3}{4}J' & -\frac{\sqrt{3}}{4}(J_1 - J_2) \\ -\frac{\sqrt{3}}{4}(J_1 - J_2) & -\frac{E_z}{2} + \frac{1}{4}J' - \frac{1}{2}(J_1 + J_2) \end{pmatrix}$$

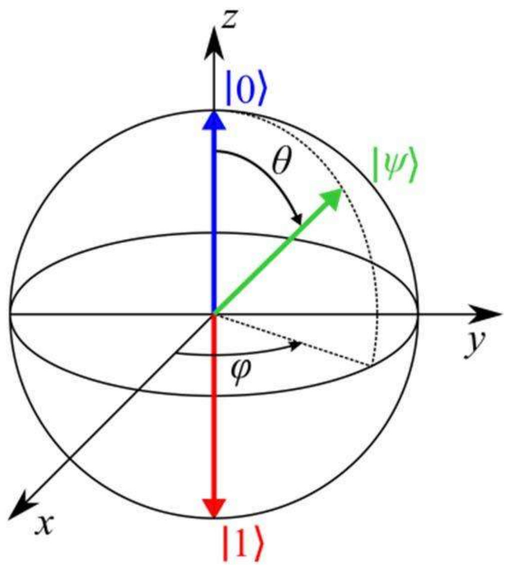
Universel set of quantum gates

- It is formed by one and two qubit gates: any set of gates to which any operation possible on a quantum computer can be reduced, that is, any other unitary operation can be expressed as a finite sequence of gates from the set.
- One qubit operations: Hadamard gate (**H**) and rotations along the Bloch sphere (**R_z**, **R_x**).



- It is indeed possible to demonstrate that a universal gate set is ***G* = {*H*,(*S*)}**, where (*S*) is a two-qubit gate in which the operation *S* is applied to the target qubit if and only if the control qubit is in the logical state $|1\rangle$, for example the CNOT gate.
- A construction of the CNOT gate using only $R_z(-\pi/2)$, H and √iSWAP gates is also feasible. (*see for example Ferraro et al. EPJ Quantum Technology 9:2 (2022)*)

Single qubit operation:
Rotations along x and z axis of the Bloch sphere



$$R_x(\phi) = \begin{pmatrix} \cos(\phi/2) & -i\sin(\phi/2) \\ -i\sin(\phi/2) & \cos(\phi/2) \end{pmatrix} \quad R_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$R_x(\phi)$ $R_z(\theta)$

$U(t) = e^{-\frac{i}{\hbar}Ht}$

Analytical Calculations

Control Sequences with pulse type, step amplitude, step time

- ❑ Unitary evolution between the i-th and j-th spins can be represented with the operator:

$$U_{ij}(t) = e^{-\frac{i}{\hbar}tJ_{ij}\mathbf{S}_i \cdot \mathbf{S}_j}$$

- ❑ *N* *c*onsecutive interactions between spin couples sequence evolution matrix *U(t)*

$$U(t) = U_N \cdot U_{N-1} \cdot ... \cdot U_1$$

Time must be non-negative and as short as possible compatibly with experimental apparatus (minimum time interval imposed = 100 ps)

To obtain $R_x(\phi)$

$$U_{J_1}(t_{J_1}) \cdot U_{J_2}(t_{J_2}) \longrightarrow R_x(\phi) = \begin{pmatrix} \cos(\phi/2) & -i\sin(\phi/2) \\ -i\sin(\phi/2) & \cos(\phi/2) \end{pmatrix}$$

$$H_{J_i} = \frac{1}{2}E_z(\sigma_1^z + \sigma_2^z + \sigma_3^z) + \frac{1}{4}J'\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{4}J_i\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_3$$

$$t_{J_1}(\phi) = \left(\frac{n}{C} - \frac{1}{\sqrt{3}}\frac{\phi}{2\pi}\frac{1}{J^{\max}}\right)h$$

$$t_{J_2}(\phi) = \left(\frac{n}{C} + \frac{1}{\sqrt{3}}\frac{\phi}{2\pi}\frac{1}{J^{\max}}\right)h,$$

$$n = \left\lceil \frac{C}{J^{\max}}\frac{1}{\sqrt{3}}\frac{\phi}{2\pi} \right\rceil.$$

E. Ferraro, M. Fanciulli, M. De Michielis, Journal of Physics Communications 2, 115022 (2018)

To obtain $R_z(\theta)$

$$U_{J_1}(t_{J_1}) \cdot U_{J'}(t_{J'}) \cdot U_{J_2}(t_{J_2}) \longrightarrow R_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$H_{J_i} = \frac{1}{2}E_z(\sigma_1^z + \sigma_2^z + \sigma_3^z) + \frac{1}{4}J'\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{4}J_i\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_3$$

$$H_{J'} = \frac{1}{2}E_z(\sigma_1^z + \sigma_2^z + \sigma_3^z) + \frac{1}{4}J'\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$t_{J_1}(\theta) = \frac{1}{C}\left[\frac{\theta}{\pi}A + sign\left(\frac{2\pi}{3} - \theta\right)B\right]\frac{h}{J^{\max}}$$

$$t_{J_2}(\theta) = t_1(\theta)$$

$$t_{J'}(\theta) = \left(2 - \frac{\theta}{\pi}\right)\frac{h}{J^{\max}}$$

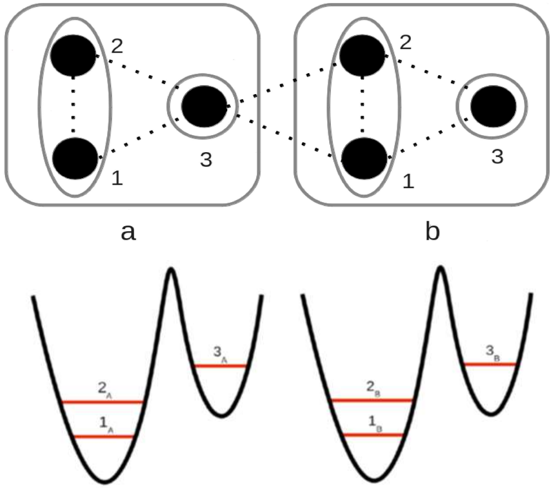
$$A = \frac{E_z}{2} + \frac{1}{8}J^{\max}$$

$$B = -E_z + \frac{1}{4}J^{\max}$$

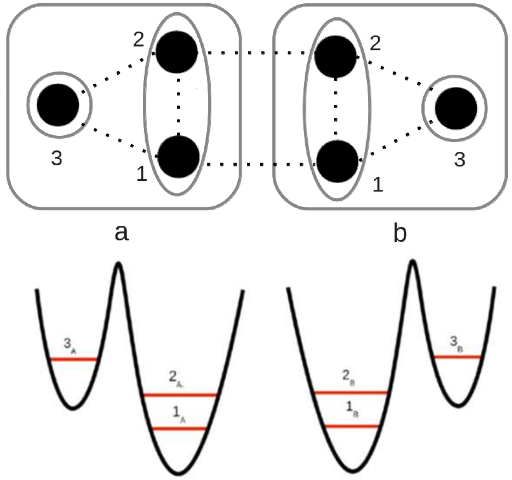
$$C = E_z + \frac{3}{4}J^{\max}.$$

E. Ferraro, M. Fanciulli, M. De Michielis, Journal of Physics Communications 2, 115022 (2018)

Effective Hamiltonian: 2 Qubits Twin Configuration



Effective Hamiltonian: 2 Qubits Mirror Configuration



$$H = H_a + H_b + H_{ab}$$

$$H_T^{eff} = H_a^{eff} + H_b^{eff} + \sum_{i=1}^{2} J_{3_a i_b} \mathbf{S}_{3_a} \cdot \mathbf{S}_{i_b}$$

$$J_{3_a 1_b} = \frac{4(t_{3_a 1_b} - J_t^{(3_a 1_b)})^2}{E_{(112,011)} - E_{(111,111)}} - 2J_e^{(3_a 1_b)}$$

$$J_{3_a 2_b} = \frac{4(t_{3_a 2_b} - J_t^{(3_a 2_b)})^2}{E_{(112,101)} - E_{(111,111)}} - 2J_e^{(3_a 2_b)}$$

$$H_M^{eff} = H_a^{eff} + H_b^{eff} + \sum_{i=1}^{2} J_{i_a i_b} \mathbf{S}_{i_a} \cdot \mathbf{S}_{i_b}$$

$$J_{1_a 1_b} = \left(\sum_{l=1}^{2} \frac{1}{\Delta E_l}\right) 4(t_{1_a 1_b} - J_t^{(1_a 1_b)})^2 - 2J_e^{(1_a 1_b)}$$

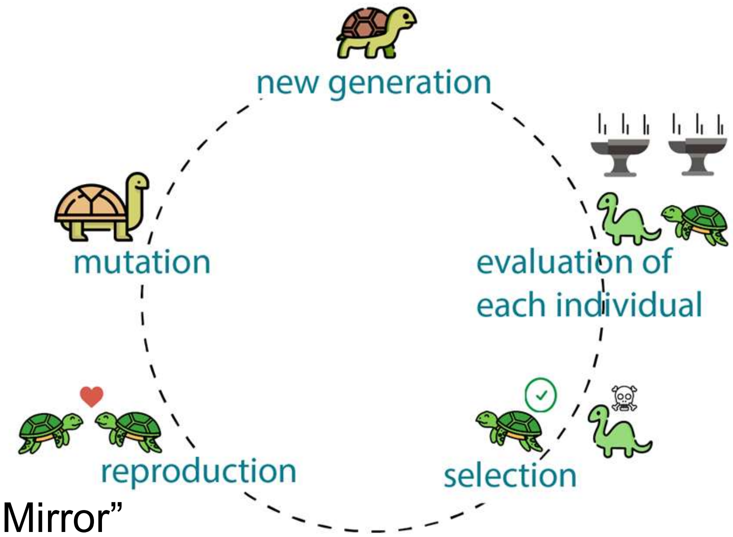
$$J_{2_a 2_b} = \left(\sum_{l=3}^{4} \frac{1}{\Delta E_l}\right) 4(t_{2_a 2_b} - J_t^{(2_a 2_b)})^2 - 2J_e^{(2_a 2_b)}$$

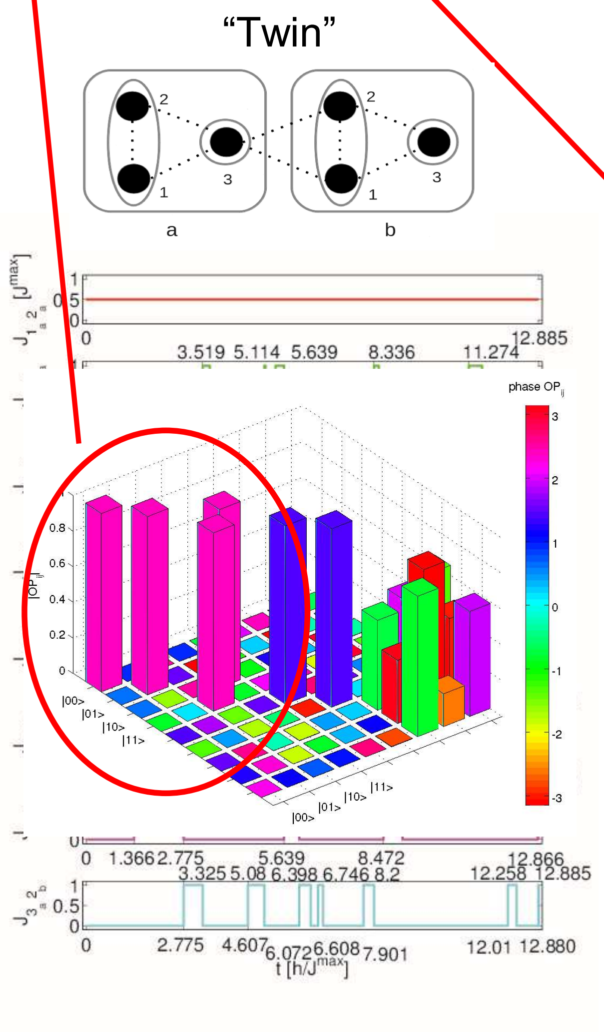
$$\Delta E_1 = E_{(011,211)} - E_{(111,111)}$$
$$\Delta E_2 = E_{(211,011)} - E_{(111,111)}$$
$$\Delta E_3 = E_{(101,121)} - E_{(111,111)}$$
$$\Delta E_4 = E_{(121,101)} - E_{(111,111)}$$

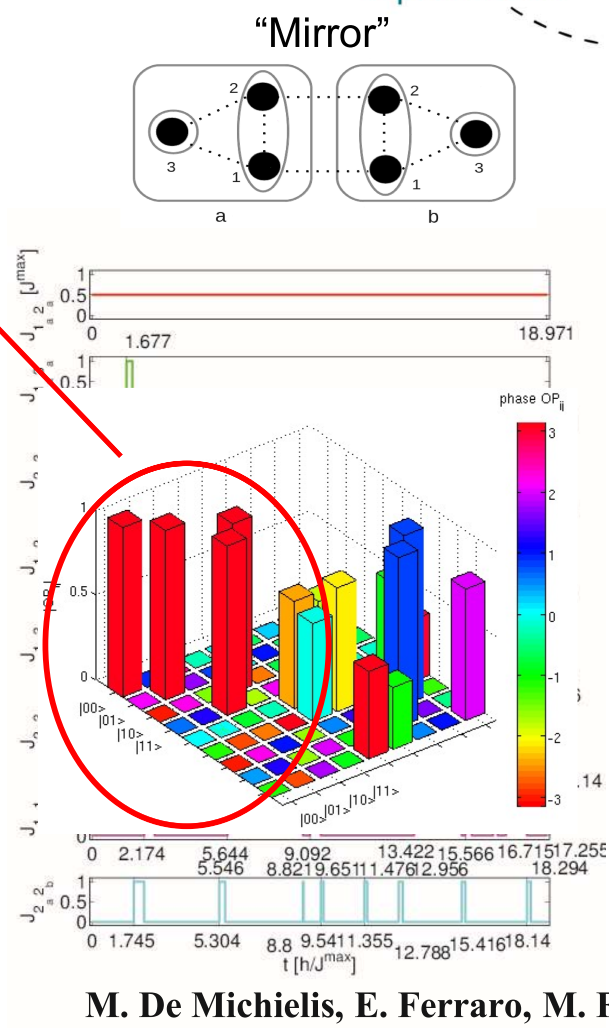
E. Ferraro, M. De Michielis, M. Fanciulli, E. Prati, Quantum Information Processing 14, 47-65 (2015)

Genetic Algorithm: CNOT Gates

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$







M. De Michielis, E. Ferraro, M. Fanciulli, E. Prati,
Journal of Physics A: Math. Theor. 48, 065304 (2015)

Outline

- ☑ Introduction: basic principle and motivations
- ☑ Quantum dot based qubit:
- Loss-DiVincenzo qubit
- Singlet-triplet qubit
- Exchange only qubit - Hybrid qubit
- ☑ Donor based qubit:
- Donor bounded e^- qubit
- ☑ Quantum dot-Donor based qubit:
- $e^-/^{31}P^+$ flip-flop qubit
- Carroll qubit
- ☑ Hybrid qubit: effective Hamiltonian model and derivation of single and double quantum operations.
- ☑ Conclusions

Conclusions

- ✓ Confinement of spins in silicon represents a fruitful platform for universal Quantum Computation.
- ✓ Spin qubit theory: derivation of effective Hamiltonian models; analytical time sequences and/or numerical strategies for the derivation of the universal set of gates composed by single and two qubit gate operations.
- ✓ Recent huge improvements towards scalability.

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